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Discrete Optimization

Branch-and-price algorithms for the solution of the multi-trip vehicle routing problem with time windows

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ABSTRACT

We investigate the exact solution of the vehicle routing problem with time windows, where multiple trips are allowed for the vehicles. In contrast to previous works in the literature, we specifically consider the case in which it is mandatory to visit all customers and there is no limitation on duration. We develop two branch-and-price frameworks based on two set covering formulations: a traditional one where columns (variables) represent routes, that is, a sequence of consecutive trips, and a second one in which columns are single trips. One important difficulty related to the latter is the way mutual temporal exclusion of trips can be handled. It raises the issue of time discretization when solving the pricing problem. Our dynamic programming algorithm is based on concept of groups of labels and representative labels. We provide computational results on modified small-sized instances (25 customers) from Solomon's benchmarks in order to evaluate and compare the two methods. Results show that some difficult instances are out of reach for the first branch-and-price implementation, while they are consistently solved with the second.

1. Introduction

1.1. Description of the problem

The Multi-Trip Vehicle Routing Problem with Time Windows (MTVRPTW) is a variant of the classical Vehicle Routing Problem with Time Windows (VRPTW) where multiple trips are allowed for vehicles within the planning time horizon. Multiple trips are beneficial to the carrier by limiting the number of vehicles and drivers necessary to the deliveries, especially in the cases where vehicle capacities, distances or time constraints naturally imply short routes.

Formally, the MTVRPTW is defined as follows. Let $[0, T]$ be the planning time horizon. Let $G = (V, A)$ be a directed graph where $V = \{0, \dots, n\}$ is the set of vertices and A is the set of arcs. Vertex 0 represents the depot and vertices $1, \dots, n$ the customers. A cost c_{ij} and a travel time t_{ij} are associated with each arc $(i, j) \in A$. Triangle inequality is assumed for times and costs. The fleet comprises U identical vehicles, with load capacity Q. For each customer $i \in \{1, \dots, n\}$ are defined a demand d_i, a time window $[a_i, b_i]$, a service time st_i and a loading time l_i. Service of customers must start within their time window; however, vehicles may arrive earlier and wait. Service times relate to handling operations at the customers, while loading times concern loading of the vehicles at the depot, before departure.

The problem is to find a set of trips of minimal cost, using at most U vehicles, and such that (i) all customers are served, (ii) two trips cannot be assigned to the same vehicle if the schedules of these trips, including loading and return to the depot, overlap, (iii) loads comply with vehicle capacities and (iv) time constraints at customers and the depot are met.

In this paper, we propose to address the MTVRPTW with two set covering formulations and branch-and-price solution schemes. As far as we know, these are the first attempts to solve this problem exactly. Branch-and-price is a variant of branch-and-bound where LP-based lower bounds are computed through column generation, because of the huge number of variables. In column generation, two problems, called the restricted master problem and the pricing problem, are solved iteratively until the pricing problem fails to find new potentially improving columns for the restricted master problem. The interested reader is referred to Desrochers, Desrosiers, and Solomon (1992), Barnhart, Johnson, Nemhauser, Savelsbergh, and Vance (1996) or Feillet (2010) for details on column generation and branch-and-price.

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1.2. Literature review

There are few published papers about multi-trip vehicle routing problems. According to Battarra, Monaci, and Vigo (2009), Fleischmann (1990) is the first to consider the multi-trip feature in a vehicle routing problem. The author proposes a two-phase heuristic based on a savings algorithm to construct the trips and a bin packing heuristic to combine them and form routes. Following this line, most of the works published on multi-trip vehicle routing problems investigate heuristic or metaheuristic approaches. A few recent exceptions however exist that address exact methods. In Sen and Bülbül (2008), an extensive survey on heuristic and metaheuristic approaches can be found. In the rest of this section we survey exact methods.

Mingozzi, Roberti, and Toth (2013) propose an exact method based on column generation to solve the Multi-Trip Vehicle Routing Problem (without time-windows). It extends previous works from the same authors on different variants of vehicle routing problems. It combines two set partitioning formulations that are successively used to compute the best possible lower bound. In the first formulation, columns are trips, in the second columns are complete vehicle routes. Note that for the former, a single constraint per vehicle is sufficient to ensure that the time horizon is respected, and that for the latter new routes are obtained by combining trips previously computed. This work seems difficult to extend for the solution of the MTVRPTW, because of the time windows that complicate the packing of trips when forming vehicle routes. In our opinion, the difficulty of this packing actually increases a lot the difficulty of the problem.

In Ceselli, Righini, and Salani (2009), a column generation algorithm is used to solve a rich vehicle routing problem in which multi-trip routes are allowed. In this scheme, columns are vehicle routes, formed by sequences of consecutive trips. The pricing problem is solved with a bidirectional dynamic programming algorithm. The authors acknowledge two important limitations of their approach. First, they approximate the loading time at the depot by assuming that it is constant. Second, the proposed column generation scheme is not embedded in a branch-and-price algorithm. Instead, a general purpose ILP solver is run with the set of generated columns to find an integer solution. As indicated by the authors, the obtained solution is not proved optimal with this method.

Recently, three papers addressed the solution of the MTVRPTW when the duration of trips is constrained (Azi, Gendreau, & Potvin, 2010; Hernandez, Feillet, Giroudeau, & Naud, 2014; Macedo, Alves, de Carvalho, Clautiaux, & Hanafi, 2011). This problem is named MTVRPTW with Limited Duration (MTVRPTW-LD). Several variants are considered, where the visit of customers is compulsory or not. All these papers proceed in two phases. The first phase consists in listing all the candidate trips. The second phase aims at constructing the vehicle routes. While these approaches could theoretically be applied for the solution of the MTVRPTW, by defining a large duration limit for the trips, they strongly rely on the fact that enumerating all the trips is possible. Hence, they cannot be applied for the MTVRPTW in practice, except on very small instances. However, they constitute a first attempt for the exact solution of the MTVRPTW and deserve to be presented in more details.

In all three papers, the first phase is addressed with the same method, initially proposed in Azi, Gendreau, and Potvin (2007) for a single-vehicle version of the problem. This method, based on dynamic programming, generates all non-dominated trips. The three papers differ in the second phase.

In Azi et al. (2010), the second phase starts with the construction of an auxiliary graph. A vertex and a time window are introduced for each trip. Arcs represent possible successions of trips. Thanks to the limited duration of trips, the size of the graph does not explode. Phase 2 can then be tackled as a variant of the VRPTW: each vehicle visits a feasible sequence of vertices in the auxiliary graph, which represents a sequence of trips in the original setting. Note that contrary to the VRPTW, all vertices need not to be covered. This problem is solved with a branch-and-price method.

In Macedo et al. (2011), the second phase relies on a time-indexed graph. The nodes of this graph correspond to discrete time instants. A time instant is an abstraction on a discrete time scale of a certain continuous time duration. This duration is called granularity. The arcs of this graph are of two types. The first arc type models the waiting time by one vehicle at the depot between two successive trips. The second arc type models the elapsed time during a given trip, which starts at a specific time instant and ends at another. A solution is then obtained by solving a flow problem on this graph. Several enhancements are proposed to limit the size of the graph and accelerate the algorithm.

In Hernandez et al. (2014), the second phase implements a branch-and-price algorithm. A set covering formulation is proposed where each column represents a single trip with a fixed schedule. In this formulation, vehicle routes are not explicitly computed; instead, a set of constraints ensures that the number of vehicles used at each time instant is compatible with the fleet size. The pricing problem simply consists in finding a convenient starting date to trips and has a pseudo-polynomial complexity. Again, time granularity is an issue in this approach.

Computationally, Macedo et al. (2011) and Hernandez et al. (2014) obtain comparable results and significantly improve upon Azi et al. (2010). However, the method proposed in Hernandez et al. (2014) clearly outperforms the one describe in Macedo et al. (2011) when the number of trips generated in the first phase increases.

1.3. Contributions of this paper

This article is devoted to the exact solution of the MTVRPTW with the following assumptions: it is mandatory to visit all customers, loading time depends on quantities delivered, and there is no *a priori* limitation on trip duration. As seen in Section 1.2, it constitutes the first study devoted to the exact solution of this problem. We propose two very different set covering formulations and branch-and-price solution approaches.

In the first formulation, following Ceselli et al. (2009), columns are vehicle routes, that is, sequences of consecutive trips. However, contrary to Ceselli et al. (2009) we develop the complete branch-and-price scheme and take into account actual loading times. With this formulation we are faced with the inconvenient that the combinatorics resulting from the succession of trips is not broken.

To circumvent this difficulty, variables of the second formulation are single trips instead of vehicle routes. This formulation follows the one previously proposed in Hernandez et al. (2014) and surveyed above. However, contrary to Hernandez et al. (2014), trips cannot be generated in advance and the pricing problem simultaneously manage the construction and the timing of the trips.

With this second formulation, the challenge is to design an efficient dynamic programming algorithm for the pricing problem. More precisely, the trip reduced cost depends on their timing, which prevents from applying traditional procedures. We propose several original concepts (groups of labels, representative label) that help dealing with this difficulty. Note that Liberatore, Righini, and Salani (2011) and Dabia, Ropke, Woensel, and De Kok (2013) face similar difficulties and investigate different solutions for the vehicle routing problem with soft time windows and the time-dependent vehicle routing problem, respectively.

1.4. Organization of this article

The paper is organized as follows. In Section 2, we present our first set covering formulation and detail the corresponding branch-and-price algorithm. Section 3 introduces the second formulation and sketches the algorithm. Further details regarding the pricing problem