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Decision Support

Optimal search for parameters in Monte Carlo simulation for derivative pricing☆

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ABSTRACT

This paper provides a novel and general framework for the problem of searching parameter space in Monte Carlo simulations. We propose a deterministic online algorithm and a randomized online algorithm to search for suitable parameter values for derivative pricing which are needed to achieve desired precisions. We also give the competitive ratios of the two algorithms and prove the optimality of the algorithms. Experimental results on the performance of the algorithms are presented and analyzed as well.

1. Introduction

Derivatives, a main category of financial instruments, play essential roles in financial markets; thus it is important to price them efficiently and accurately. However, most complex derivatives are not known to have closed-form formulas for their prices; consequently, they are priced using numerical methods such as Monte Carlo simulation. Originated from studies in physics, Monte Carlo methods have been applied in finance (Boyle, 1977), many studies of which have focused on problems of pricing derivatives via Monte Carlo methods (e.g., Belomestny, Bender, & Schoenmakers, 2009; Kimura & Shinohara, 2006). For example, some papers focus on coping with financial derivatives with early exercise opportunities, such as American options and Bermuda options (e.g., Bacinello, Biffis, Millossovich, 2009; Broadie and Detemple, 1996; Ibáñez, 2004; Kan, Reesor, Whitehead, and Davison, 2009; Liu, 2010; Longstaff and Schwartz, 2001; Rogers, 2002). Other studies concern methods for pricing derivatives under different assumptions for underlying assets, such as Lévy processes (e.g., Becker, 2010; Dingeç & Hörmann, 2012; Ribeiro & Webber, 2006) and Carr–Geman–Madan–Yor processes (e.g., Ballotta and Kyriakou, 2014).

Standard Monte Carlo simulation has a simple bound of $O(1/\sqrt{N})$ for the standard error for N paths (Hull, 2009). However, when Monte Carlo simulation is combined with other approximation techniques or is used to price complicated financial instruments, analytical analysis of its convergence is difficult to obtain (e.g., Dingeç & Hörmann, 2012; Glasserman & Li, 2005; Kan et al., 2009; Liu, 2010; Longstaff & Schwartz, 2001; Miao & Lee, 2013). In addition, computing the standard error after sampling each additional path is not applicable for some approximation techniques, such as least-squared Monte Carlo (Liu, 2010; Longstaff & Schwartz, 2001), because each additional path necessitates recalculating the prices of all other paths. Aggravating the absence of analytical convergence in most simulation methods, there has been little research on general frameworks for searching for parameter values (e.g., the number of paths) in Monte Carlo simulation that are required to achieve desired precisions while minimizing running time or other computational resources.

The problem of searching for an object in an unknown environment is central to many areas of computer science and operational research; many variants of search problems have been studied (e.g., Baeza-Yates, Culberson, & Rawlins, 1993; Chrobak, Kenyon, Noga, & Young, 2008; Kao, Reif, & Tate, 1993). Online algorithms are usually adopted to deal with the problem of searching in an unknown environment as they can process the sequence of requests piece-by-piece in a serial fashion without having the entire input available from the start. Competitive analysis was developed to analyze online algorithms; the competitive ratio of an online algorithm for an

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optimization problem is the worst-case ratio between the cost of the solution found by the online algorithm and the cost of an optimal solution.

Viewing our problem as a problem of searching in an unknown environment, we provide a general framework for searching parameter space in Monte Carlo simulation, which is orthogonal to the previous studies in this field. Specifically, we propose two optimal online algorithms (one deterministic and one randomized) to search for suitable parameter values for derivative pricing which are needed to achieve desired precisions. We also conduct a competitive analysis of the proposed online algorithms and give the algorithm parameters that minimize the competitive ratios in order to yield the best performance; finally, the optimality of the algorithms are also proved. Furthermore, experimental results present the performance of the algorithms and show that the algorithms can be applied to the pricing of various kinds of options, i.e., vanilla call and put options, American options, and Bermudan options. The selected pricing methods and options in the experiments are for demonstration purposes; our proposed search algorithms can be applied to any effective simulation pricing method in a straightforward manner.

This paper is organized as follows. In Section 2, some preliminaries for derivative pricing are introduced. In Section 3, we describe the proposed deterministic and randomized online algorithms and provide the competitive ratios and the proofs of their optimality. In Section 4, experimental results on the performance of the algorithms are presented and analyzed. In Section 5, we conclude the paper and provide some limitations of our work and future research directions.

2. Preliminaries

2.1. Background financial knowledge

Derivatives are popular financial instruments, whose values depend on other more fundamental financial assets called the underlying assets. The most common underlying assets include stocks, bonds, commodities, and currencies; the most common types of derivatives are forwards, futures, options, and swaps. Hereafter, the underlying asset is assumed to be a stock and the derivative is assumed to be an option for simplicity of discussion.

Vanilla options give their owners the rights to buy or sell the underlying stocks for the exercise price X and have no other unusual features. European-style vanilla options allow holders to exercise the options only at the maturity date T with the payoff

$$\mathcal{P}(T) = \max(\theta \cdot (S_T - X), 0), \tag{1}$$

where S_T denotes the stock price at time T, $\theta = 1$ is for call options, and $\theta = -1$ is for put options.

American options allow holders to exercise the options prior to the maturity date T. The value to exercise an American option at time $t\,(0 \le t < T)$ is $\theta \cdot (S_t - X)$. The holders exercise the options early if it is more beneficial than keeping them.

Bermudan options are a hybrid of American and European options, where the buyer has the right to exercise at the date of expiration, and on certain specified dates that occur between the purchase date and the maturity date.

A risk-neutral measure is heavily adopted in the pricing of financial derivatives due to the fundamental theorem of asset pricing (Schachermayer, 2009). This theorem implies that in a complete market, the price of a derivative is the discounted expected value of the future payoff under the unique risk-neutral measure. Particularly, a European-style option's value at time 0 equals the discounted expected payoff at time T (Harrison & Pliska, 1981), i.e.,

$$e^{-rT}\mathrm{E}[\mathcal{P}(T)], \tag{2}$$

where r is the risk-free rate.

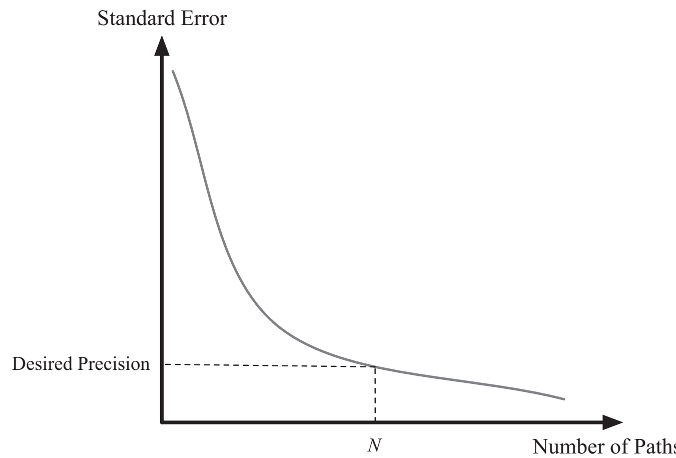

Fig. 1. A Decreasing function for the standard error of the estimate. For an effective simulation method, the standard error of the estimate should be a decreasing function of the number of paths.

2.2. Monte Carlo simulation for derivative pricing

In finance, the geometric Brownian motion is widely used to model the stock price process (e.g., Black & Scholes, 1973), which can be represented as the following differential form under the risk-neutral probability:

$$dS_t = rS_t dt + \sigma S_t dW_t, \tag{3}$$

where r is the risk-free rate, σ is the volatility of the stock prices, and the random variable dW_t is the standard Brownian motion. Eq. (3) has the following solution (Shreve, 2004):

$$S_t = S_0 e^{(r-\sigma^2/2)t + \sigma W_t}. \tag{4}$$

When Monte Carlo methods are used to price derivatives, risk-neutral valuation (Schachermayer, 2009) is usually adopted. Numerous paths are sampled to obtain the expected payoff in a risk-neutral world and then discount this payoff at the risk-free rate (see Eq. (2)). We now use the pricing of a European-style vanilla call option to demonstrate the standard procedure of Monte Carlo simulation. This option provides a payoff at time T as in Eq. (1) with $\theta = 1$. Assuming the stock price follows the geometric Brownian motion in Eq. (4), the option can be priced via the following steps:

1. Sample a normal random variable $W_T \sim N(0, T)$ and plug it into Eq. (4) to obtain S_T.
2. Calculate the payoff $\max(S_T - X, 0)$ from the option.
3. Repeat steps 1 and 2 for d paths to obtain d sample values of the payoff from the option.
4. Calculate the mean of the sample payoffs to obtain an unbiased estimate of the expected payoff in a risk-neutral world.
5. Discount the expected payoff at the risk-free rate to obtain an estimate of the option price.

The precision of the above estimated price in general depends on the number of paths; it is usually measured by the standard error of the estimate (i.e., the standard deviation of the sample means' estimates of a population mean). The standard Monte Carlo simulation has a simple bound of $O(1/\sqrt{N})$ for the standard error for N paths. However, when Monte Carlo simulation is combined with other approximation techniques or is used to price complicated financial instruments, analytical analysis of its convergence is difficult to obtain. Even so, for an effective simulation method, in general, the standard error should be a decreasing function of the number of paths (see Fig. 1).