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A multi-stage stochastic programming approach in master production scheduling

Ersin Körpeoğlu [a], Hande Yaman [b], M. Selim Aktürk [b,*]

[a] Tepper School of Business, Carnegie Mellon University, 5000 Forbes Avenue, Pittsburgh, PA 15213, USA
[b] Department of Industrial Engineering, Bilkent University, 06800 Ankara, Turkey

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ABSTRACT

Master Production Schedules (MPS) are widely used in industry, especially within Enterprise Resource Planning (ERP) software. The classical approach for generating MPS assumes infinite capacity, fixed processing times, and a single scenario for demand forecasts. In this paper, we question these assumptions and consider a problem with finite capacity, controllable processing times, and several demand scenarios instead of just one. We use a multi-stage stochastic programming approach in order to come up with the maximum expected profit given the demand scenarios. Controllable processing times enlarge the solution space so that the limited capacity of production resources are utilized more effectively. We propose an effective formulation that enables an extensive computational study. Our computational results clearly indicate that instead of relying on relatively simple heuristic methods, multi-stage stochastic programming can be used effectively to solve MPS problems, and that controllability increases the performance of multi-stage solutions.

1. Introduction

Master Production Schedules (MPS) are widely used by manufacturing facilities to handle production and scheduling decisions. In current industry practice, the MPS produces production schedules in a finite planning horizon, assuming infinite capacity, fixed processing times, and deterministic demand.

Our study is motivated by the following application. The largest auto manufacturer in Turkey recently introduced a new multi-purpose vehicle to the market. The company installed a single production line with a limited production capacity and dedicated it to this particular model. Since the production facilities are flexible, the processing times could be altered or controlled (albeit at higher manufacturing cost) by changing the machining conditions in response to demand changes. As this model is new, the company generated different demand scenarios for each time period. One of the important planning problems was to develop a master production schedule to determine how many units of this new model would be produced in each time period along with the desired cycle time (or equivalently, the optimal processing times) to satisfy the demand and available capacity constraints, with the aim of maximizing the total profit. This plan will be used in their Enterprise Resource Planning (ERP) system as an important input to the materials management module to explode the component requirements and generate the required purchase and shop floor orders for the lower level components.

Motivated by this application, we consider the following problem setting. We have a single work center with controllable processing times. The work center produces a single product type with a given price, manufacturing cost function, processing time upper bound, i.e., processing time with minimum cost, and maximum compressibility value. As in the case of MPS, we have a finite planning horizon. The orders arrive at the beginning of each period and the products are replenished at the end of the period. There is an additional cost of postponement if the replenishment cannot be done by the end of the period.

The demand of the first period is assumed to be known with certainty prior to scheduling. However, the demand of the other periods are uncertain; possible scenarios for demand realizations and their associated probabilities are known. In our MPS calculations, the number of units of demand is defined in terms of the multiples of a base unit. Therefore, a job represents the amount of one base unit. Our objective is to maximize the total expected profit by deciding how many units to produce, when to produce, and how to produce them, i.e., the required processing times.

Our aim in this paper is to question the basic assumptions of MPS regarding infinite capacity, fixed processing times, and deterministic demand, and to propose a new approach that overcomes to an extent the disadvantages caused by these assumptions and is computationally efficient. In the remaining part of this section, we briefly summarize the existing work on MPS, scheduling with controllable processing times, and multi-stage stochastic programming. We conclude the section with an example that motivates our study.

* Corresponding author. Tel.: +90 3122901360; fax: +90 3122664054.
E-mail addresses: ekorpeog@andrew.cmu.edu (E. Körpeoğlu), hyaman@bilkent.edu.tr (H. Yaman), akturk@bilkent.edu.tr (M. Selim Aktürk).

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The classical approach for generating Master Production Schedules assumes known demands, infinite capacity, and fixed processing times. In the current literature on MPS, the demand uncertainty is ignored during the schedule generation. As a result, the main research focuses on the length of the frozen time period, i.e., the number of periods in which production scheduling decisions are not altered even when demand realizations turn out to be different than the estimates. A longer frozen time period is less responsive to demand changes, but creates less nervousness, while a shorter one acts oppositely. Studies by Sridharan et al. (1987) and Tang and Grubbström (2002) are examples that consider the effect of the length of the frozen zone on production and inventory costs. Based on his industry experience, Vieira (2006) points out that the real complexity involved in making a master plan arises when capacity is limited and when products have the flexibility of being produced at different settings. As opposed to the current literature, we consider different demand scenarios with given probabilities along with the controllable processing times and finite capacity of the available production resources while generating the schedule.

There are several instruments that can be used to control processing times. For example, in computer numerical control (CNC) machining operations, the processing time can be controlled by changing the feed rate and the cutting speed. As the cutting speed and/or the feed rate increases, the processing time of the operation compresses at an additional cost that arises due to increased tooling costs, as discussed in Gurel and Akturk (2007). This scenario results in a strictly convex cost function for compression. Cheng et al. (2006) study a single machine scheduling problem with controllable processing times and release dates. They assume that the cost of compression is a linear function of the compression amounts. Leyvand et al. (2010) provide a unified model for solving single-machine scheduling problems with due date assignment and controllable job-processing times. They assume that the job-processing times are either a linear or a convex function of the amount of a continuous and nonrenewable resource that is to be allocated to the processing operations. In our study, we define the compression cost function $f(y) = \kappa \cdot y^{a/b}$ as discussed in Kayan and Akturk (2005), where y is the amount of compression, a and b are two positive integers such that $a > b > 0$, and κ is a positive real number. We use a nonlinear compression cost function as opposed to a linear cost function as widely used in the literature, since it reflects the law of diminishing marginal returns.

A review of scheduling with controllable processing times can be found in Shabtay and Steiner (2007), in which they also summarize possible applications in a steel mill and in an automated manufacturing environment in addition to the automotive industry example that we have discussed above. As far as our problem is concerned, controllable processing times may constitute a flexibility in capacity since the maximum production amount can be increased by compressing the processing times of jobs with, of course, an additional cost. Thus, this scenario brings up the trade-off between the revenue gained by satisfying an additional demand and the amount of compression cost. The value of controllable processing times becomes even more evident during economic crises, since they allow companies to adjust their production quantities to meet the immediate demand that varies significantly during the planning horizon more effectively.

Stochastic programming uses mathematical programming to handle uncertainty. Although deterministic optimization problems are formulated with parameters that are known with certainty, in real life it is difficult to know the exact value of every parameter during planning. Stochastic programming handles uncertainty assuming that probability distributions governing the data are known or can be estimated. The goal here is to maximize the expectation of some function of the decisions and random variables. Such models are formulated, analytically or numerically solved, and then analyzed in order to provide useful information to a decision-maker.

Two-stage stochastic programs are the most widely used versions of stochastic programs. The decision maker takes some action in the first stage, after which a random event occurs that affects the outcome of the first-stage decision. A recourse decision can then be made in the second stage to compensate for any negative effect that might have been experienced as a result of the first-stage decision. A detailed explanation of stochastic programming, its applications, and solution techniques can be found in Birge and Louveaux (1997) and a survey of two-stage stochastic programming is given in Schultz et al. (1996). Using more than one stage in decision making is also utilized in robust optimization. Atamturk and Zhang (2007) apply two-stage robust optimization to network flow and design problems. They give a numerical example that explains the benefit of using two stages instead of a single one.

In multi-stage stochastic programming, decisions are made in several decision stages instead of two. At each stage, a different decision is made or recourse action is taken. Multi-stage stochastic programming models may yield better results than two-stage models since they incorporate data as they become available, and hence enable a more certain environment for decision making. On the other hand, they are generally more difficult to solve than their two-stage counterparts, therefore, their applications are rare.

In the context of production planning, the early work of Holt et al. (1956) explicitly considers uncertain demand and flexible workforce capacity, whereas Charnes et al. (1958), Bookbinder and Tan (1988) and Orcun et al. (2009) use chance constraints to address problems with uncertain demand. Furthermore, Peters et al. (1977), Escudero et al. (1993), Voss and Woodruff (2006), Karabuk (2008) and Higle and Kempf (2011) apply multi-stage stochastic programming to production planning. Balibek and Koksalan (2010) apply a multi-objective multi-stage stochastic programming approach for the public-debt management problem. Guan et al. (2006) study the uncapacitated lot-sizing problem and Ahmed et al. (2003) study the capacity expansion problem with uncertain demand and cost parameters. Huang and Ahmed (2009) provide analytical bounds for the value of multi-stage stochastic programming over the two-stage approach for a general class of capacity planning problems under uncertainty. To the best of our knowledge, there is no study in the literature that applies multi-stage stochastic programming to master production scheduling. Stochastic programming problems are generally considered difficult (Dyer and Leen, 2006).

When the uncertain parameters evolve as a discrete-time stochastic process with finite probability space, the uncertainty can be represented with a scenario tree; Fig. 1.1 depicts an example.

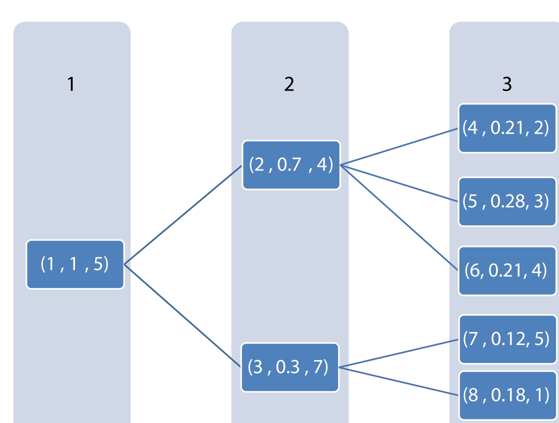

Fig. 1.1. A scenario tree for three periods.