European Journal of Operational Research 202 (2010) 114–121

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Production, Manufacturing and Logistics

The rectilinear distance Weber problem in the presence of a probabilistic line barrier

Mustafa S. Canbolat *, George O. Wesolowsky

DeGroote School of Business, McMaster University, 1280 Main St. West, Hamilton, ON, Canada L8S 4M4

ARTICLE INFO

Article history:
Received 25 July 2008
Accepted 24 April 2009
Available online 5 May 2009

Keywords:
Planar facility location
Barriers to travel
Probabilistic barriers
Line barrier
The Weber problem

ABSTRACT

This paper considers the problem of locating a single facility in the presence of a line barrier that occurs randomly on a given horizontal route on the plane. The objective is to locate this new facility such that the sum of the expected rectilinear distances from the facility to the demand points in the presence of the probabilistic barrier is minimized. Some properties of the problem are reported, a solution algorithm is provided with an example problem, and some future extensions to the problem are discussed.

1. Introduction

Consider locating a facility on the plane to serve a finite set of existing demand points with different demand levels. One objective would be to find a location such that the sum of the distances from the facility to the demand points is minimum. This is the classical Weber problem. Difficulties in solution methodologies occur when there are restrictions on location.

Most of the restricted planar location problems that are studied in the literature fall in one of the three categories. The first category considers forbidden regions where no facility placement is allowed or possible; however travelling through these regions is not restricted. For an overview of planar location problems with forbidden regions, the reader is referred to Hamacher and Nickel (1995).

The second category considers congested regions. A congested region is a region where placement of a facility is forbidden but travelling through is possible with some penalty.

The third category deals with barrier regions. Mountains, lakes, military zones, existing facilities, railroads, highways, etc. can be given as examples of barrier regions where neither travel through these regions nor placement in a region is possible. Although facility location problems in the presence of barrier regions have more practical relevance than general facility location problems, they have not been given much attention until lately, due to the computational complexities associated with these problems. Table 1 is an overview and classification of the facility location problems in the presence of barrier regions studied in the literature.

Research in the area starts with Katz and Cooper (1981) which is the first paper that considered the Weber problem with Euclidean distances and barrier regions. The authors discussed the problem with one circular barrier and showed that the problem had a non-convex objective function. A heuristic based solution approach is proposed with no guarantee of the global optimum. Some properties of the problem were later analyzed by Klamroth (2004) who suggested dividing the feasible region into some convex regions where the objective function is convex in each region. The number of such convex regions is bounded by $O(N^2)$ where N is the number of demand points. When N increases, construction of these convex regions becomes cumbersome hence is not desired. To get over this difficulty, Bischoff and Klamroth (2007) proposed a genetic algorithm based solution to the problem.

Aneja and Parlar (1994) considered the Weber problem with Euclidean distances and convex or non-convex polyhedral barriers. The solution procedure proposed by the authors generates some candidate locations using simulated annealing and, for each candidate location, a visibility graph is constructed to find the shortest path network. The shortest path between any candidate location and existing facility location is found using Dijkstra's algorithm, which finds shortest paths on networks in a polynomial time.

Butt and Cavalier (1996) developed an algorithm that finds some local optima to the Euclidean distance Weber problem in the presence of some polyhedral barriers. The authors proposed a decomposition of the feasible region into subregions in which shortest barrier distance between two points remain constant throughout the region. The problem with this approach is that the boundaries of the subregions are generally nonlinear. Klamroth (2001a) suggested a different decomposition approach by applying visibility grids to the same problem to overcome this difficulty.

* Corresponding author. Tel.: +1 905 525 9140x26361; fax: +1 905 521 8995.
E-mail address: canbolat@mcmaster.ca (M.S. Canbolat).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2009.04.023

Table 1
Facility location in the presence of barriers literature overview.

	Distance	Objective	Interaction	Facility shape	Barrier shape	Barrier type	Result
Katz and Cooper (1981)	Euclidean	Minisum	User–facility	Point	Circular	Fixed	Heuristic
Larson and Sadiq (1983)	Rectilinear	Minisum	User–facility	Point	Arbitrary	Fixed	Optimal
Batta et al. (1989)	Rectilinear	Minisum	User–facility	Point	Arbitrary	Fixed	Optimal
Aneja and Parlar (1994)	Euclidean	Minisum	User–facility	Point	Arbitrary	Fixed	Heuristic (SA)
Butt and Cavalier (1996)	Euclidean	Minisum	User–facility	Point	Convex polygonal	Fixed	Local optimal
Hamacher and Klamroth (2000)	Block	Minisum	User–facility	Point	Convex polygonal	Fixed	Optimal
Klamroth (2001a)	Any	Minisum or center	User–facility	Point	Convex polygonal	Fixed	Optimal
Klamroth (2001b)	Euclidean	Minisum	User–facility	Point	Line with passages	Fixed	Optimal
Dearing et al. (2002)	Rectilinear	Center	User–facility	Point	Convex polygonal	Fixed	Optimal
Dearing and Segars (2002a)	Rectilinear	Minisum or center	User–facility	Point	Arbitrary	Fixed	Optimal
Dearing and Segars (2002b)	Rectilinear	Minisum or center	User–facility	Point	Arbitrary	Fixed	Optimal
Klamroth and Wiecek (2002)	Any	Minisum	User–facility	Point	Line with passages	Fixed	Pareto optimal
Savas et al. (2002)	Rectilinear	Minisum	User–user and user–facility	Finite	Arbitrary	Variable	Heuristic
Wang et al. (2002)	Rectilinear	Minisum	User–facility	Finite or point	Rectangular	Fixed	Optimal
McGarvey and Cavalier (2003)	Euclidean	Minisum	User–facility	Point	Convex polygonal	Fixed	Optimal
Nandikonda et al. (2003)	Rectilinear	Center	User–facility	Point	Arbitrary	Fixed	Optimal
Klamroth (2004)	Euclidean	Minisum	User–facility	Point	Circular	Fixed	Optimal
Dearing et al. (2005)	Block	Minisum or center	User–facility	Point	Convex polygonal	Fixed	Optimal
Frieß et al. (2005)	Euclidean	Center	User–facility	Point	Convex polygonal	Fixed	Optimal (experimental)
Bischoff and Klamroth (2007)	Euclidean	Minisum	User–facility	Point	Convex polygonal	Fixed	Heuristic (GA)
Kelachankuttu et al. (2007)	Rectilinear	Minisum	User–user and user–facility	Rectangular	Rectangular	Variable	Multiple optimal
Sarkar et al. (2007)	Rectilinear	Center	User–facility	Finite	Arbitrary	Variable	Optimal
This paper	Rectilinear	Minisum	User–facility	Point	Line	Probabilistic	Optimal

A modified version of the Big Square Small Square (BSSS) method was proposed by McGarvey and Cavalier (2003) for the Euclidean distance Weber problems with barriers. The BSSS method is a Branch and Bound (B&B) technique that divides the feasible region into square subregions and produces either a global optimal solution or a solution within a very small tolerance of the global optimum. The method was originally proposed by Hansen et al. (1981) for locating obnoxious facilities. In this method non-convex polygonal barrier regions can also be considered.

Larson and Sadiq (1983) is a seminal work that first considered using the rectilinear distances for facility location problems in the presence of barrier regions. The authors examined the rectilinear distance p-median problem on the plane with polyhedral barrier regions and defined a special structured grid that contains nodes and edges. They discovered that this set of nodes provides a finite dominating set of solution points for the problem.

These fundamental results motivated some researchers who continued working on the same problem to provide some extensions. First, Batta et al. (1989) extended the work by considering both convex forbidden regions and arbitrarily shaped barriers. Second, findings in the Ph.D. thesis by Segars (2000), and their extensions were published by Dearing and Segars (2002a,b). In the first paper, using the visibility idea, the authors showed that the barriers can be modified without affecting the objective value, thus allowing some non-convex barrier shapes to be equivalent to convex ones. Also the feasible region can be reduced by this modification, and it can be decomposed into rectangular cells and these rectangular cells can be partitioned into convex domains where the distance functions are convex and methods from convex optimization can be used to solve the problem over these convex domains. The second paper discusses this solution methodology and provides an example, which gives an optimal solution on the nodes as in Larson and Sadiq (1983) also in a convex cell. This is important because one does not have to restrict herself to nodes of the network to get an optimal value.

Similar results, based on smart ways of decomposition of the planar feasible region, are provided by Dearing et al. (2002) for rectilinear center location problems with polyhedral barriers, who proposed an algorithm for the problem by considering a finite number of candidate sets called dominating sets to find the optimal location. Later, these results are extended by Dearing et al. (2005) using block norm distances in place of rectilinear distances. This work is also an extension to Hamacher and Klamroth (2000) who first considered block norm distances for the Weber problem with barriers.

There is also another body of research extending the studies of Larson and Sadiq (1983) and Batta et al. (1989). The first study is Savas et al. (2002), who proposed a model for the finite size facility placement problem in the presence of some barriers under the rectilinear distance norm. Interaction between a facility and demand points is handled through the facility's server point which is located on the facility's boundary. The demand points also interact with each other. The finite size facility, which has a fixed size and an arbitrary shape, acts as a barrier against the flow among the demand points. The authors provided concavity results for a facility location with fixed orientation and for a facility orientation with a fixed location. Possible heuristics are suggested for simultaneous location and orientation decisions.

A special case of this problem in which the supply facility and the demand facilities have rectangular shapes was discussed by