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An approximate algorithm for prognostic modelling using condition monitoring information

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ABSTRACT

Established condition based maintenance modelling techniques can be computationally expensive. In this paper we propose an approximate methodology using extended Kalman-filtering and condition monitoring information to recursively establish a conditional probability density function for the residual life of a component. The conditional density is then used in the construction of a maintenance/replacement decision model. The advantages of the methodology, when compared with alternative approaches, are the direct use of the often multi-dimensional condition monitoring data and the on-line automation opportunity provided by the computational efficiency of the model that potentially enables the simultaneous condition monitoring and associated inference for a large number of components and monitored variables. The methodology is applied to a vibration monitoring scenario and compared with alternative models using the case data.

1. Introduction

Condition based maintenance (CBM) scheduling and replacement applications involve the utilisation of condition monitoring (CM) information and prognostic models of an operational component's state or condition. In many studies, the underlying state of the component is modelled as the residual life (RL), or time remaining until failure, at discrete CM sampling points during its operational life. However, when developing a prognostic model for a given on-line automated application, two major considerations are not effectively catered for in the existing literature on CBM. The first issue is the incorporation of multi-dimensional CM information without resorting to approximation and data reduction. The second issue is an ability to rapidly process a large amount of data without the use of numerical integration routines, as required when using the existing techniques in the literature. Established techniques such as proportional hazards modelling, Kumar and Westberg (1997), Makis and Jardine (1991), and Love and Guo (1991), non-linear probabilistic stochastic filtering, Wang and Christer (2000), Wang (2002, 2006, 2007), and Wang and Zhang (2008), hidden Markov models, Bunks and Mccarthy (2000), Makis and Jiang (2003) and Zhou et al. (2010), can be computationally expensive to apply simultaneously to a large number of individually monitored components and multi-dimensional CM variables.

The Kalman-filter is a useful approach for discrete time state estimation using stochastically related indicatory information; see Jazwinski (1970). Efficient closed form updating and prediction equations are easily established to recursively determine a posterior distribution for the underlying state using the indicatory information. As such, it is a useful technique for on-line CM applications involving a large number of components and multi-dimensional monitored variables when limited time and computational resources are available, Christer et al. (1997). The standard Kalman-filter can be derived within the framework of a general non-linear filter when the system and observation dynamics evolve linearly and the model errors are assumed to be independent and follow 0-mean Gaussian white noise processes; see Jazwinski (1970) and Harvey (1989). However, in reality, these assumptions rarely hold. There are a number of varieties of extended Kalman-filter (EKF) available in the literature on stochastic state estimation techniques, Liu et al. (2010). EKF's are designed to enable the application of variations on the standard Kalman filtering methodology to linearised versions of non-linear systems of equations. The linearisation is achieved using Taylor expansions of the state and observation equations. The order of the filter is dependent on the number of terms in the Taylor expansions that

Abbreviations: CBM, condition based maintenance; CM, condition monitoring; RL, residual life; EKF, extended Kalman filter; MLE, maximum likelihood estimation; AIC, Akaike information criterion.

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are included in the linearised equations, with a standard first order EKF utilizing only the first term.

In this paper, a semi-deterministic form of the EKF is adapted specifically for applications involving CM information where, the underlying state that is the focus of the prediction effort is defined as the RL of an individual component. We assume that the component is subject to a single dominant mode of deterioration and failure. The deterministic element of the EKF process is designed to facilitate the exact relationship between realisations of the actual underlying RL at sequential CM points throughout the components lifetime. We then illustrate the application of a vibration-based version of the model using an example and compare the predictive abilities of the model and the associated replacement decisions with those obtained using a survival analysis model and a non-linear probabilistic filter-based model, Wang (2002). When constructing EKF algorithms for conditional RL prediction, the same principles will apply when using different types of CM information, such as oil analysis data, with the only difference being the specification of the relationship between the observed CM information and the underlying RL.

2. A semi-deterministic extended Kalman filter

2.1. Problem description

We consider a plant subject to a CM process with CM information available at discrete CM data sampling points. By sampling we mean to take the CM data at regular or irregular intervals. Define $\mathbf{Y}_i$ and $\mathbf{X}_i$ as the CM information and the hidden state of the plant at a particular sampling time t_i where i is the ith CM since new. Our interest is to estimate the hidden state $\mathbf{X}_i$ from available CM information. We suppose that $\mathbf{Y}_i$ is multi-dimensional of the same or different nature, which could be, for example, the vibration and temperature at t_i, but $\mathbf{X}_i$ is a single dimensioned variable to describe the plant state which is not directly observable. However in order to keep the standard EFK presentation, we still use a matrix notation for $\mathbf{X}_i$. There is no restriction to the type of CM data and the type of the plant, but a fundamental assumption here is that $\mathbf{Y}_i$ and $\mathbf{X}_i$ are stochastically related with each other through a conditional probabilistic density function of $\mathbf{Y}_i|\mathbf{X}_i$ which implies that $\mathbf{Y}_i$ is a function of $\mathbf{X}_i$ with some random noise. Since the plant state degradation is a stochastically decreasing process so the assumption requires that $\mathbf{Y}_i$ is also stochastically decreasing or increasing. In other words, a general trend must be present in the measured CM data. The exact relationship between $\mathbf{Y}_i$ and $\mathbf{X}_i$ will be problem specific and has to be decided at the model fitting stage. For now we assume that such a conditional probabilistic density function of $\mathbf{Y}_i|\mathbf{X}_i$ exists.

2.2. The EKF model

Considering a discrete time process, the evolution of a general-state random vector is described using the non-linear function

$$\mathbf{X}_{i+1} = f(\mathbf{X}_i = \mathbf{x}_i), \quad (1)$$

where $\mathbf{x}_i$ is the unknown realisation of the state at the ith discrete CM time point, at time t_i. Eq. (1) is called the 'state transition' equation. At the ith discrete CM time point, we describe the relationship between an observed information vector and the underlying state using the non-linear function, h, as

$$\mathbf{Y}_i = h(\mathbf{X}_i = \mathbf{x}_i) + \mathbf{E}_i, \quad (2)$$

where, $\mathbf{Y}_i$ is the observation vector at t_i and the measurement errors are normally distributed as $\mathbf{E}_i \sim \mathrm{N}(\mathbf{0}, \mathbf{R}_i)$ where $\mathbf{R}_i$ is the covariance matrix. Clearly $\mathbf{Y}_i|\mathbf{X}_i$ follows a normal distribution with mean $h(\mathbf{x}_i)$ and covariance matrix $\mathbf{R}_i$. Eq. (2) is called the 'observation' equation. It is noted that the observation is modelled as a function of the state, and therefore, is not used exogenously. Eq. (1) is deterministic while Eq. (2) is stochastic because of $\mathbf{E}_i$ and as such Eqs. (1) and (2) define a semi-deterministic Kalman filter.

The first step in applying the extended Kalman filtering methodology is to linearise the functions f and h in the system of equations given by (1) and (2) as noted in Liu et al. (2010). At the ith discrete time point, we define $\boldsymbol{\alpha}_{i|i} = \mathbf{E}[\mathbf{X}_i|\Im_i]$ as the expectation of $\mathbf{X}_i$ that is conditioned on the observation history available until that point; $\Im_i = \{\mathbf{Y}_1 = \mathbf{y}_1, \mathbf{Y}_2 = \mathbf{y}_2, \ldots, \mathbf{Y}_i = \mathbf{y}_i\}$. We also define $\boldsymbol{\alpha}_{i+1|i} = E[\mathbf{X}_{i+1}|\Im_i]$ as a one-step prediction of $\mathbf{X}_{i+1}$ that is again conditioned on $\Im_i$. The non-linear functions f and h are linearised approximately as, Liu et al. (2010),

$$f(\mathbf{X}_i = \mathbf{x}_i) \approx f(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i}) + f'(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i})(\mathbf{x}_i - \boldsymbol{\alpha}_{i|i}), \quad (3)$$

$$h(\mathbf{X}_i = \mathbf{x}_i) \approx h(\mathbf{X}_i = \alpha_{i|i-1}) + h'(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1})(\mathbf{x}_i - \boldsymbol{\alpha}_{i|i-1}). \quad (4)$$

Using these approximations, the state transition expression and the relationship between the observation vector and the underlying state are expressed as

$$\mathbf{X}_{i+1} = f'(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i})\mathbf{x}_i + \mathbf{u}_i, \quad (5)$$

$$\mathbf{Y}_i = h'(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1})\mathbf{x}_i + \mathbf{E}_i + \mathbf{w}_i \quad (6)$$

with $\mathbf{u}_i = f(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i}) - f'(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i})\boldsymbol{\alpha}_{i|i}$ and $\mathbf{w}_i = h(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1}) - h'(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1})\boldsymbol{\alpha}_{i|i-1}$, where f' and h' are the differentials of the functions f and h respectively.

Assuming that the initial values for the underlying state, $\mathbf{X}_0 = \mathbf{x}_0$, and the associated covariance matrix, $\mathbf{P}_0$, are known or can be estimated from the data, the Kalman filtering algorithm can then be applied directly to the linearised system given by Eqs. (5) and (6) when observing new information. The recursive algorithm incorporates prediction and updating steps. At the ith discrete CM time point, the equation for updating the mean estimate of the underlying state with the availability of the observed information vector, $\mathbf{Y}_i = \mathbf{y}_i$, is

$$\begin{aligned}\boldsymbol{\alpha}_{i|i} &= \boldsymbol{\alpha}_{i|i-1} + k_i\left[\mathbf{y}_i - h'(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1})\boldsymbol{\alpha}_{i|i-1} - \mathbf{w}_i\right] \\ &= \boldsymbol{\alpha}_{i|i-1} + k_i\left[\mathbf{y}_i - h(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1})\right],\end{aligned} \quad (7)$$

where, the well known Kalman gain function (see Harvey (1989)) is

$$k_i = \mathbf{P}_{i|i-1}h'\left(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1}\right)^{\mathrm{T}}\left[h'\left(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1}\right)\mathbf{P}_{i|i-1}h'\left(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1}\right)^{\mathrm{T}} + \mathbf{R}_i\right]^{-1}. \quad (8)$$

For a semi-deterministic EKF, using the original state transition expression given by Eq. (1), a one-step forecast of the mean state vector is simply

$$\boldsymbol{\alpha}_{i+1|i} = f'\left(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i}\right)\boldsymbol{\alpha}_{i|i} + \mathbf{u}_i = f\left(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i}\right). \quad (9)$$

At the ith time point, the covariance matrix is updated using

$$\mathbf{P}_{i|i} = \mathbf{P}_{i|i-1} - k_i h'\left(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i-1}\right)\mathbf{P}_{i|i-1} \quad (10)$$

and a one-step prediction of the covariance matrix is

$$\mathbf{P}_{i+1|i} = f'(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i})\mathbf{P}_{i|i}f'\left(\mathbf{X}_i = \boldsymbol{\alpha}_{i|i}\right)^{\mathrm{T}}. \quad (11)$$

This concludes the description of the semi-deterministic EKF algorithm for general discrete time state-vector and observation-vector processes. Higher order terms in the Taylor expansions of the system and observation equations can also be incorporated; see the appendix for details.

3. Residual life prediction using vibration information

3.1. Modelling the CM process

In this section, we tailor the EKF methodology to a CM scenario involving vibration monitoring and RL estimation for operational