European Journal of Operational Research 211 (2011) 208–212

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Short Communication

# A note on "Price discount based on early order commitment in a single manufacturer-multiple retailer supply chain"

Huachun Xiong, Jinxing Xie *, Miao Wang

*Department of Mathematical Sciences, Tsinghua University, Beijing 100084, China*

ARTICLE INFO

*Article history:*
Received 20 August 2009
Accepted 28 September 2010
Available online 20 October 2010

*Keywords:*
Supply chain management
Early order commitment
0–1 programming
Supply chain coordination
Wholesale price discount contract

ABSTRACT

In a recent paper by Xie et al. [Xie, J., Zhou, D., Wei, J.C., Zhao, X., 2010. Price discount based on early order commitment in a single manufacturer-multiple retailer supply chain. European Journal of Operational Research 200, 368–376], the authors have studied the early order commitment (EOC) strategy for a decentralized, two-level supply chain consisting of a single manufacturer and multiple retailers. They fail to provide an algorithm to determine the optimal EOC periods to minimize the total supply chain cost. This note proposes a polynomial-time algorithm to find the optimal solutions, and provides a new set of sufficient conditions under which the wholesale price discount scheme coordinates the whole supply chain.

© 2010 Elsevier B.V. All rights reserved.

## 1. Introduction

In a recent paper, Xie et al. (2010) provided an analytical model to quantify the effects of early order commitment (EOC) strategy on the performance of a two-level supply chain consisting of a single manufacturer and $N$ independent retailers. Under EOC strategy, retailer $i$ ($i = 1,2,\ldots,N$) places her order $x_i$ periods in advance, where $x_i$ is called the EOC period for retailer $i$. In order to minimize the expected holding and shortage cost per period for the whole supply chain, Xie et al. (2010) proposed the following optimization problem:

$$\operatorname*{Min}_{0\leqslant x_i\leqslant L_0+1} SC(x) = r_0\sqrt{\sum_{i=1}^{N}\left(\frac{\sigma_i}{1-\rho_i}\right)^2\sum_{j=L_i+x_i+2}^{L_i+L_0+2}(1-\rho_i^j)^2} + \sum_{i=1}^{N} r_i\frac{\sigma_i}{1-\rho_i}\sqrt{\sum_{j=1}^{L_i+x_i+1}(1-\rho_i^j)^2}, \quad (1)$$

where $x = (x_1,x_2,\ldots,x_N)$ are decision variables, and $L_0 > 0$, $r_0 > 0$, $L_i > 0$, $d_i > 0$, $\sigma_i > 0$, $0 < \rho_i < 1$ and $r_i > 0$ ($i = 1,2,\ldots,N$) are known parameters (please refer to Xie et al., 2010 for details). They failed to provide an algorithm to find an optimal solution to Problem (1). In Section 2 of this note, we propose a polynomial-time algorithm to find the optimal solutions.

Xie et al. (2010) also proposed a wholesale price discount scheme to induce the retailers to practice EOC strategy and identified a set of sufficient conditions under which the scheme coordinates the whole supply chain. In Section 3 of this note, we provide a new set of sufficient conditions which also leads to supply chain coordination.

## 2. An optimal algorithm

In Theorem 1 of Xie et al. (2010), they identified an amazing characteristic for the optimal solutions of Problem (1): the EOC period $x_i$ for each retailer $i$ should be either 0 or $L_0 + 1$. Therefore, we can define $y_i = (L_0 + 1 - x_i)/(L_0 + 1)$, where $y_i \in \{0,1\}$ ($i = 1,2,\ldots,N$), and $y_i = 0$ means that retailer $i$ uses EOC policy, and $y_i = 1$ means that retailer $i$ does not use EOC policy. After the variable redefinition, the objective function $SC(x)$ in Problem (1) can be expressed as a function of $y = (y_1,y_2,\ldots,y_N)$:

$$\overline{SC}(y) = \left(\sum_{i=1}^{N} a_i y_i\right)^{\frac{1}{2}} - \sum_{i=1}^{N} b_i y_i + c, \quad (2)$$

* Corresponding author. Tel.: +86 10 6278 7812; fax: +86 10 62785847.
*E-mail address:* jxie@math.tsinghua.edu.cn (J. Xie).

0377-2217/$ - see front matter © 2010 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2010.09.043

where

$$a_i = \left(\frac{\sigma_i r_0}{1-\rho_i}\right)^2 \sum_{j=L_i+2}^{L_i+L_0+2} (1-\rho_i^j)^2, \tag{3}$$

$$b_i = \frac{r_i \sigma_i}{1-\rho_i}\left(\sqrt{\sum_{j=1}^{L_0+L_i+2}(1-\rho_i^j)^2} - \sqrt{\sum_{j=1}^{L_i+1}(1-\rho_i^j)^2}\right), \tag{4}$$

$$c = \sum_{i=1}^{N} r_i \frac{\sigma_i}{1-\rho_i}\sqrt{\sum_{j=1}^{L_0+L_i+2}(1-\rho_i^j)^2}. \tag{5}$$

Since $a_i$, $b_i$, $c$ are constants independent of the decision variables, Problem (1) is equivalent to the following 0–1 programming problem:

$$\min_{y_i\in\{0,1\}} \quad f(y) = \left(\sum_{i=1}^{N} a_i y_i\right)^{\frac{1}{2}} - \sum_{i=1}^{N} b_i y_i. \tag{6}$$

Now consider the following class of 0–1 programming problems:

$$\min_{y_i\in\{0,1\}} \quad f(y) = \left(\sum_{i=1}^{N} a_i y_i\right)^{p} - \left(\sum_{i=1}^{N} b_i y_i\right)^{q}, \tag{7}$$

where $a_i > 0$, $b_i > 0$, $0 \leqslant p \leqslant 1$ and $q \geqslant 1$. Obviously, Problem (6) is a special case of Problem (7) with $p = 1/2$, $q = 1$. For Problem (7), we have the following theorem.

**Theorem 1.** *Suppose that N pairs of positive numbers* $(a_i, b_i)$, $i = 1,2,\dots,N$, *satisfy* $a_1/b_1 \geqslant a_2/b_2 \geqslant a_3/b_3 \geqslant \cdots \geqslant a_N/b_N$.

(*a*) *If* $p = q = 1$, *then there exists a binary vector* $y = (y_1, y_2, \dots, y_N)$ *minimizing* (7) *and satisfying the following property: If* $y_j = 0$ *for some j* $(1 \leqslant j \leqslant N)$, *then* $y_i = 0$ *for any* $1 \leqslant i < j$.

(*b*) *If* $0 \leqslant p < 1$ *and* $q \geqslant 1$, *or* $0 \leqslant p \leqslant 1$ *and* $q > 1$, *then the binary vector* $y = (y_1, y_2, \dots, y_N)$ *minimizing* (7) *should satisfy the following property: If* $y_j = 0$ *for some j* $(1 \leqslant j \leqslant N)$, *then* $y_i = 0$ *for any* $1 \leqslant i < j$.

**Proof.** Part (a) is obviously true. For Part (b), we only provide a proof for the case of $0 \leqslant p < 1$ and $q \geqslant 1$, since the proof for the other case is similar.

Suppose $y$ is a binary vector minimizing (7) with $y_j = 0$ for some $j$ $(1 \leqslant j \leqslant N)$ and $y_i = 1$ for some $1 \leqslant i < j$. Denote $y'$ as a binary vector where $y'_i = 0$ and $y'_k = y_k$ for all $k \neq i$. By contradiction, we only need to prove that $f(y') < f(y)$.

Denote $A = \sum_{k\neq i,j} a_k y_k$ and $B = \sum_{k\neq i,j} b_k y_k$. By definition of $f(y)$, $f(y') < f(y)$ is equivalent to

$$(A + a_i)^p - A^p > (B + b_i)^q - B^q. \tag{8}$$

To prove Inequality (8), we choose $y''$ such that $y''_i = y''_j = 1$ and $y''_k = y_k$ for all $k \neq i, j$. Since $y$ is an optimal solution of (7), we have $f(y) \leqslant f(y'')$, which is equivalent to

$$(A + a_i + a_j)^p - (A + a_i)^p \geqslant (B + b_i + b_j)^q - (B + b_i)^q. \tag{9}$$

Since $a_i > 0$ and $b_i > 0$ for all $i = 1,2,\dots,N$, Inequality (9) implies Inequality (8) if the following inequality holds:

$$\frac{(A + a_i)^p - A^p}{(A + a_i + a_j)^p - (A + a_i)^p} > \frac{(B + b_i)^q - B^q}{(B + b_i + b_j)^q - (B + b_i)^q}. \tag{10}$$

Now we prove Inequality (10). Consider a function $g(u) = u^p$. By Mean Value Theorem, there exists a $\xi \in (A + a_i, A + a_i + a_j)$ such that

$$p\xi^{p-1} = g'(\xi) = \frac{(A + a_i + a_j)^p - (A + a_i)^p}{a_j}. \tag{11}$$

Similarly, there exists a $\eta \in (A, A + a_i)$ such that

$$p\eta^{p-1} = g'(\eta) = \frac{(A + a_i)^p - A^p}{a_i}. \tag{12}$$

Clearly, $0 < \eta < \xi$. This, together with the fact that $g'(u) = pu^{p-1}$ $(0 \leqslant p < 1)$ is strictly decreasing with respect to $u$ $(u > 0)$, implies that $p\eta^{p-1} > p\xi^{p-1}$. Therefore, by Eqs. (11) and (12), we have

$$\frac{(A + a_i)^p - A^p}{a_i} > \frac{(A + a_i + a_j)^p - (A + a_i)^p}{a_j},$$

which is equivalent to

$$\frac{(A + a_i)^p - A^p}{(A + a_i + a_j)^p - (A + a_i)^p} > \frac{a_i}{a_j}. \tag{13}$$

Download English Version:

https://daneshyari.com/en/article/480841

Download Persian Version:

https://daneshyari.com/article/480841

Daneshyari.com