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Interfaces with Other Disciplines

Portfolio rebalancing model using multiple criteria

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ABSTRACT

In order to achieve greater flexibility in portfolio selection, transaction cost, short selling and higher moments should be considered, and actual transactions should be reflected. In this paper, five portfolio rebalancing models, with consideration of transaction cost and consisting of some or all criteria, including risk, return, short selling, skewness, and kurtosis, are compared to determine the important design criteria for a portfolio model. Two examples are used to perform simulated transactions, and the results indicate that the investment strategy of 'buy and hold' does not produce better returns for all the portfolios in the first example, and the models with higher moments or adopting short selling strategy perform better while rebalancing in the second example.

1. Introduction

The mean–variance model of the portfolio selection, originally developed by Markowitz (1952), is one of the most important portfolio selection models in contemporary financing. Subsequently, numerous refinements have been proposed to improve the performances of investment portfolios, resulting in a large number of research focused on risk versus return, and diversification in investment strategies (Yusen et al., 2000). Markowitz's mean–variance model involves only two criteria; however, other criteria are taken into account by most investors, each with their own portfolio considerations.

Multiple criteria decision aids (MCDA) have been more frequently applied in daily life (Brans, 2004), as decisions formed upon thoughts of multiple criteria can satisfy a variety of issues and reach compromises that lead to greater harmony. In practical application, diverse criteria and preferences are usually resulted from different decision making strategies, as investors are generally concerned with more than two criteria. An investment strategy depends on the investors' preferences (Steuer and Na, 2003), and the selection of a proper portfolio is critical in achieving the investors' various objectives. Hallerbach and Spronk (1997) pointed out that most models fail to incorporate the multidimensional nature of portfolio selection issues, and only outlined a framework view focused on portfolio management. Spronk et al. (2005) proposed seven criteria for portfolio considerations; however, the discuss remains in the conceptual stage and has not yet been implemented.

As suggested, portfolio selection must consist of more criteria than only risk and return in order to provide investors with additional choices. For example, maximizing the criteria of skewness in a portfolio would result in better return (Konno and Suzuki, 1995; Yoshimoto, 1996). Chunhachinda et al. (1997) indicated that skewness values of efficient mean–variance-skewness portfolios are higher than those of efficient mean–variance portfolios. In the probability theory and statistics, skewness is a measure of asymmetry within the probability distribution of random variables. In finance, the criterion of skewness in a portfolio must be considered.

Empirical findings suggest that the incorporation of skewness into an investor's portfolio may result in an improved optimal portfolio (Prakash et al., 2003; Joro and Na, 2006; Yu et al., 2008; Li et al., 2010). Excluding skewness may lead to an inefficient portfolio (Leung et al., 2001). Sun and Yan (2003) pointed out that positively skewed portfolios of individual stocks would exhibit a greater extent of persistence. From practical evidence, most distributions of stock market returns are not normal distributions, and are characterized by significant skewness and kurtosis (Tang and Shum, 2003; Gondzio and Grothey, 2007). Most researches address the importance of considering skewness and kurtosis when evaluating the performance of a portfolio (Zakamouline and Koekebakker, 2009; Díaz et al., 2009) or risk management (Hong et al., 2009). However, short selling is rarely included in the above research when considering higher moments.

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White (1990) pointed out that the rationale of short selling is highly speculative, and thus, is more appropriate for use with low investment risks. From a normative viewpoint, Kwan (1997) argued that short selling has good potential to improve a portfolio's risk-return trade-off. Although short selling has inherent high risks, it is adopted by most investors to obtain interest arbitrage (Angel et al., 2003). However, in order to avoid high risks, the proportion of short selling is expected to be minimized. Jacobs et al. (2005) presented a portfolio optimization scheme that includes short selling, with an algorithm that shows how an investor can sell short and buy long. However, they only focused on algorithms for short selling, without mentioning other criteria.

Arnott and Wanger (1990) suggested that ignoring transaction costs would lead to an inefficient portfolio; whereas, adding transaction costs would assist decision makers to better understand the behavior of an efficient frontier (Sadjadi et al., 2004). Fang et al. (2006) proposed a portfolio rebalancing model with transaction costs, but did not consider short selling. Transaction costs play a crucial role in transactions (Zhang and Zhang, 2009; Chen and Wang, 2008). Moreover, constructing more realistic models, which incorporate market frictions, are also important (Choi et al., 2007; Kozhan and Schmid, 2009).

As decision makers must consider many factors in order to meet the requirements of real transactions, developing a comprehensive portfolio selection model that includes additional factors and the preferences of the investors, is a very important issue (Branke et al., 2009). In addition, multi-period portfolio selection models, which can periodically generate rebalancing if the situation are subject to future changes, is essential (Fang et al., 2006; Bertsimasa and Pachamanovab, 2008; Yu et al., 2010; Çanakoğlu and Özekici, 2010). To our knowledge, researches that consider transaction cost, short selling, and higher moment are few. Therefore, this paper aims to propose five portfolio rebalancing models that consider the transaction cost and consist of some or all criteria, including risk, return, short selling, skewness, and kurtosis, in order to determine important design criteria for a portfolio model that can be presented to investors.

The remainder of this paper is organized as follows. First, a mean–variance model of a portfolio selection is briefly reviewed in Section 2. In Section 3, five models are developed for portfolio selection to assist investors to solve problems. Fuzzy multi-objective programming is employed to address the issues of the five proposed portfolio models. Section 4 illustrates the proposed methods, and demonstrates two numerical examples. Finally, the conclusions and future research directions are presented in Section 5.

2. The mean–variance model

The mean–variance model (Markowitz, 1952) addresses portfolio selection problems and determines the composition for a portfolio of n securities, which minimizes risks while achieving a given level of expected returns, as follows:

$$\text{Min} \quad \sigma_p = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n} \sigma_{ij} w_i w_j, \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} r_i w_i \geqslant \mu, \tag{2}$$

$$\sum_{i=1}^{n} w_i = 1, \tag{3}$$

$$w_i \geqslant 0, \quad i = 1, 2, \ldots, n,$$

where n is the number of available securities; w_i is the investment portion in i securities for $i = 1, \ldots, n$; r_i is the return on securities i; μ is the expected portfolio return; σ_i^2 is the variance of the return on securities i; and σ_{ij} is the covariance between the returns of securities i and j. The first constraint expresses the requirements of a portfolio return, while the second is the budget constraint. From $w_i \geqslant 0$, we can assume that short selling is not allowed. On the contrary, short selling is taken into consideration in the proposed model for multi-periods. Therefore, in the proposed models, w_i becomes an unrestricted sign regarding short selling.

3. The proposed models

Five multi-objective models, which consist of criteria including return, risk, proportion of short selling, skewness, and kurtosis, are evaluated. The four rebalancing multi-objective models with short selling are the MVS (mean, variance, and short selling), the MVS_S (mean, variance, short selling, and skewness), the MVS_K (mean, variance, short selling, and kurtosis), the MVS_SK (mean, variance, short selling, skewness, and kurtosis), while the model MV (mean and variance) does not involve short selling. The five models are presented and compared in order to determine which models perform the best.

(1) The model MVS consists of four objectives, as shown in Eqs. (4)-(7), namely, the maximization of portfolio return, the minimization of portfolio risk as measured by the portfolio variance, the minimization of the short selling proportion of the portfolio, and the minimization of transaction cost. In order to consider short selling, w_i is an unrestricted sign, which is decomposed into $w_i^+ - w_i^-$. The details of the model MVS are as follows:
Model MVS:

$$\text{Max} \sum_{i=1}^{n} r_i (w_i^+ - w_i^-), \tag{4}$$

$$\text{Min} \sum_{i=1}^{n} (w_i^+ - w_i^-)^2 \sigma_i^2 + \sum_{i=1}^{n} \sum_{j=1(j\neq i)}^{n} \sigma_{ij} (w_i^+ - w_i^-)(w_j^+ - w_j^-), \tag{5}$$

$$\text{Min} \sum_{i=1}^{n} w_i^-, \tag{6}$$