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An occurrence of multiple projections in DEA-based measurement of technical efficiency: Theoretical comparison among DEA models from desirable properties

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Abstract

This study discusses nine desirable properties that a measure of technical efficiency (TE) needs to satisfy from the perspective of production economics and optimization. Seven data envelopment analysis (DEA) models are theoretically compared from a viewpoint of nine TE criteria. All the seven DEA models suffer from a problem of multiple projections even though a unique projection for efficiency comparison is one of the nine desirable properties. Furthermore, all the DEA models violate the property on aggregation of inputs and outputs. Thus, the seven DEA models do not satisfy all desirable TE properties. In addition, the comparison provides us with the following guidelines: (a) The additive model violates all desirable TE properties. (b) Russell measure and SBM (=ERGM) perform as well as RAM as a non-radial measure. If we are interested in strict monotonicity, the two models outperform the other DEA models including RAM. In contrast, if we are interested in translation invariance, RAM is better than Russell measure and SBM (=ERGM). (c) The radial measures (CCR and BCC) have the property of linear homogeneity. (d) The CCR model is useful for measuring a frontier shift among different periods. (e) If a data set contains a negative value, RAM becomes a DEA model to handle the negative value because it has the property of translation invariance. After examining the desirable TE properties, this study proposes a new approach to deal with an occurrence of multiple projections. The proposed approach includes a test to examine an occurrence of multiple projections, a mathematical expression of a projection set, and a selection process of a unique reference set as the largest one covering all the possible reference sets.

Keywords: Data envelopment analysis; Technical efficiency; Linear programming

1. Introduction

The measurement of technical efficiency (TE) was axiomatically proposed by Debreu (1951, 1959). Following Debreu (1951), Farrell (1957) used an activity analysis approach with matrix inversions. Based upon the advice of Alan Hoffman (an OR researcher) who was one of the commentators in the paper of Farrell (1957), a linear programming (LP) formulation for TE was first proposed in the paper by Farrell and Fieldhouse (1962). Unfortunately, almost no attention had been given to their work on TE for a long time. The TE measurement was extensively investigated after Charnes et al. (1978) proposed data envelopment analysis (DEA). The contribution of previous research efforts on DEA was in the bibliography of Tavares (2002) that listed more than 3000 previous contributions from 1978 to 2001.

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Generally speaking, the previous research proposed two groups of DEA models. One of the two was referred to as "radial measures", while the other was "non-radial measures." The radial models (e.g., CCR: Charnes et al., 1978 and BCC: Banker et al., 1984) belong to the Debreu–Farrell measure. An important feature of this group is that TE is measured by an efficiency score in the objective function of the LP formulation. Meanwhile, the non-radial measures (e.g., an additive model: Charnes et al., 1985) determine TE by examining only these slacks, because each non-radial measure does not have any efficiency score in the objective function. This type of measures belongs to Pareto–Koopmans measure (Koopmans, 1951; Russell, 1985, p.109).

A difference between the Debreu–Farrell type of TE measure and the CCR/BCC type of TE measure exist in the objective of these LP formulations. The latter models incorporate slacks in the objective function of these LP formulations with a non-Archimedean small number. In contrast, the Debreu–Farrell measure does not have such slacks in the objective. Hence, the TE measure, using CCR and BCC, is due to not only the Debreu–Farrell measure but also the Pareto–Koopmans measure.

In the history of DEA, we proposed several different types of non-radial models for TE-based performance evaluation. Charnes et al. (1985) first proposed an additive model as a non-radial model. Then, Charnes et al. (1982, 1983) linked the additive model to a multiplicative model by changing a data set into a natural logarithm. Cooper et al. (1999) also proposed a non-radial DEA model referred to as "a range-adjusted measure (RAM)" that was an extension of the additive model (Cooper et al., 2000a, 2001). Meanwhile, Tone (2001) proposed another type of non-radial model referred to as slacks-based measure (SBM). In a similar manner, Pastor et al. (1999) proposed enhanced Russell graph measure (ERGM), as a non-radial measure, that incorporated the analytical feature of Russell measure into the framework of the SBM. Bardhan et al. (1996b) first formulated the Russell measure. The measure provided a linkage between the radial and non-radial models so that it could eliminate a need for a separate treatment of input-oriented and output-oriented efficiency measures in the radial-based TE framework.[1]

Admitting the contribution of these previous DEA studies, we are always wondering which DEA model should be used for performance analysis. Can a user apply any DEA model to any performance evaluation? The inquiry indicates research necessity to investigate several TE criteria, based upon which we can determine an appropriate DEA model. Of course, we know the existence of previous DEA research efforts (e.g., Cooper et al., 1999) that have investigated desirable TE properties and have compared several DEA models. However, these previous comparisons covered part of important radial and non-radial DEA models from partial perspective of TE criteria. Furthermore, these previous studies did not provide mathematical proofs on the desirable TE criteria in a unified manner. Hence, the purpose of this study is to investigate both what criteria (properties) are important for the TE measurement and how to select an appropriate DEA model under such different TE criteria. All the TE criteria need to be mathematically proved in a unified analytical synthesis. Moreover, the review on DEA models identifies that all DEA models examined in this study suffer from an occurrence of multiple projections even though a unique projection for efficiency comparison is one of the desirable properties. Hence, this study proposes a new approach for both identifying an occurrence of multiple projections and selecting a reference set among all possible reference sets.

The remaining structure of this article is organized as follows: Section 2 summarizes desirable TE properties obtained from previous DEA studies. Section 3 reviews radial and non-radial models from the perspective of TE criteria. Section 4 summarizes previous research efforts on the TE criteria. Section 5 proposes a standard model that covers the analytical features of all radial and non-radial DEA models. The standard model is used in mathematical proofs for examining whether each DEA model satisfies or violates the TE criteria. Section 6 examines theoretically seven DEA models. We propose a new approach to identify an occurrence of multiple projections in Section 7. We also discuss how to select a reference set among all possible reference sets. A conclusion and future extensions are summarized in Section 8.

2. Criteria for technical efficiency (TE)

The TE measurement has the following desirable properties:

Homogeneity (Banker et al., 1984; Russell, 1985; Dmitruk and Koshevoy, 1991; Blackorby and Russell, 1999): An output-based TE measure should be homogeneous of degree one in output quantities, while an input-based TE measure should be homogeneous of degree minus one in input quantities. For example, if we double all the input quantities, then the input-based TE measure should be cut in half.

Strict monotonicity (Banker et al., 1984; Russell, 1985; Dmitruk and Koshevoy, 1991; Blackorby and Russell, 1999; Cooper et al., 1999): A TE measure should be non-decreasing in output quantities and non-increasing in input quantities, along with an efficient output (input) vector.

[1] For example, we can handle imprecise (fuzzy) data by the radial and non-radial DEA models. Furthermore, we can control returns to scale by adding the upper and lower bounds on the sum of lambdas in these DEA models. This study does not discuss these DEA models because theoretical exploration discussed in this study can be applied to them. See, Cooper et al. (2007a, pp. 4–5) that explained an origin of the Russell measure.