European Journal of Operational Research 237 (2014) 15–28

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

Implicit depot assignments and rotations in vehicle routing heuristics

Thibaut Vidal [a,b,*], Teodor Gabriel Crainic [c], Michel Gendreau [d], Christian Prins [a]

[a] Institut Charles Delaunay – LOSI, Université de Technologie de Troyes, 12 rue Marie Curie, BP 2060, 10010 Troyes Cedex, France
[b] CIRRELT et Département d'informatique et de recherche opérationnelle, Université de Montréal, Montréal H3C 3J7, Canada
[c] CIRRELT et Département de management et technologie, École des sciences de la gestion, UQAM, Montréal H3C 3P8, Canada
[d] CIRRELT et Département de mathématiques et génie industriel, École Polytechnique, Montréal H3C 3A7, Canada

ARTICLE INFO

Article history:
Received 16 November 2012
Accepted 28 December 2013
Available online 7 January 2014

Keywords:
Vehicle routing
Dynamic programming
Local search
Multi-depot
Fleet mix
Unlimited fleet

ABSTRACT

Vehicle routing variants with multiple depots and mixed fleet present intricate combinatorial aspects related to sequencing choices, vehicle type choices, depot choices, and depots positioning. This paper introduces a dynamic programming methodology for efficiently evaluating compound neighborhoods combining sequence-based moves with an optimal choice of vehicle and depot, and an optimal determination of the first customer to be visited in the route, called *rotation*. The assignment choices, making the richness of the problem, are thus no more addressed in the solution structure, but implicitly determined during each move evaluation. Two meta-heuristics relying on these concepts, an iterated local search and a hybrid genetic algorithm, are presented. Extensive computational experiments demonstrate the remarkable performance of these methods on classic benchmark instances for multi-depot vehicle routing problems with and without fleet mix, as well as the notable contribution of the implicit depot choice and positioning methods to the search performance. New state-of-the-art results are obtained for multi-depot vehicle routing problems (MDVRP), and multi-depot vehicle fleet mix problems (MDVFMP) with unconstrained fleet size. The proposed concepts are fairly general, and widely applicable to many other vehicle routing variants.

1. Introduction

Vehicle Routing Problems (VRP) with combined assignment choices, such as multi-depot and mixed-fleet settings, appear prominently in many applications related to transportation, production planning, robotics, maintenance, health care or emergency relief. These combinatorial optimization problems require two levels of decisions, related respectively to the sequencing of visits to customers into routes, and the assignment of customers to some global resources, such as depots or vehicle types. Heuristics and meta-heuristics that rely on separate optimization procedures for addressing each aspect, e.g. separate families of local searches, large neighborhoods or crossovers to work on the order of visits, the depot choices or the vehicle types, may overlook a wide range of potential solution refinements involving joint changes in the sequencing and assignment decisions, e.g. swapping two customers and in the meantime changing the vehicle or the depot assigned to the routes. Thus, most advanced meta-heuristics combine these decisions within purposeful optimization procedures to achieve notable performance gains (see Prins, 2009b, for example), though the number of combined solution changes tends to become computationally expensive to investigate.

To contribute towards addressing this challenge, this paper proposes a new bidirectional dynamic programming approach to optimally manage the choices of vehicle, depot, and first customer visited in a route, the so-called optimal *rotation*, directly at the level of route evaluations in vehicle routing heuristics. We thus introduce a new Local Search (LS) in which the neighborhoods are solely based on customer-visits relocations and arc exchanges, while dynamic programming-based route evaluation functions produce optimal depot placements, choices and rotations for each alternative route. Since several advanced meta-heuristics for vehicle routing problems require a Split algorithm to optimally segment a solution represented as a giant tour into several routes, we also derive an advanced *Split* algorithm with compound vehicle assignments, depot choices and rotations. The proposed enhanced procedures work with the same computational complexity as the classic ones from the literature. Thus, the additional capability we introduce does not lead to any additional computational overhead.

As a proof of concept, these methodologies are integrated and tested within two meta-heuristics: a simple multi-start Iterated Local Search (ILS) similar to the one of Prins (2009a) for the

* Corresponding author at: Institut Charles Delaunay – LOSI, Université de Technologie de Troyes, 12 rue Marie Curie, BP 2060, 10010 Troyes Cedex, France. Tel.: +33 3 25 71 58 54; fax: +33 3 25 71 56 49.

E-mail addresses: thibaut.vidal@cirrelt.ca (T. Vidal), TeodorGabriel.Crainic@cirrelt.ca (T.G. Crainic), michel.gendreau@cirrelt.ca (M. Gendreau), christian.prins@utt.fr (C. Prins).

http://dx.doi.org/10.1016/j.ejor.2013.12.044
0377-2217/

capacitated VRP, and a more elaborate Hybrid Genetic Search with Advanced Diversity Control (HGSADC) similar to the one of Vidal, Crainic, Gendreau, Lahrichi, and Rei (2012, 2013, 2014). Two specific problems are investigated, the multi-depot fleet mix problem requiring combined decisions on assignments to vehicles types and depot along with sequencing choices, and the classic multi-depot VRP with unlimited fleet. Extensive computational experiments, on small- and large-scale benchmark instances with up to 960 customers demonstrate the remarkable performance of the proposed meta-heuristics, as well as the notable contribution of the combined neighborhoods to the search performance. In particular, the HGSADC built on these compound neighborhoods outperforms all current methods on multi-depot vehicle routing variants with or without mixed-fleet, and with unlimited fleet size. All known optimal solutions, with up to 150 customers, are systematically found.

To facilitate the presentation, we first introduce in Section 2 the problems, notations and variants considered in our experiments. Sections 3 and 4 describe the proposed methodology for optimally managing depot, vehicle choices and rotations within route evaluations, and presents the advanced Split method. The integration of these procedures into a neighborhood-based and a population-based meta-heuristic is discussed in Section 5. The computational experiments are reported in Section 6, and Section 7 concludes.

2. Vehicle routing problems and variants

Vehicle Routing Problems (VRP) aim to design least cost vehicle routes to service geographically dispersed customers (Toth & Vigo, 2002; Cordeau, Laporte, Savelsbergh, & Vigo, 2007; Golden, Raghavan, & Wasil, 2008; Laporte, 2009; Vidal, Crainic, Gendreau, & Prins, 2013c). Emphasis is still growing on this family of problems after 50 years of research, mostly because of their major economic impact in many application fields, but also because of the considerable amount of problem variants that must be dealt with to adequately address practical settings. Practical applications, indeed, lead to a variety of problem *attributes* that complement the classic VRP model, and seek to account for customer requirements (e.g., schedules, consistency), network and vehicle characteristics (mixed fleet, multiple depots), and driver's needs (working hour regulations, lunch breaks) among others.

Two attributes especially, *mixed-fleet* and *multi-depot*, are recurrent in many large-scale logistics applications. The problem combining these attributes, known as the Multi-Depot Vehicle Fleet Mix Problem (MDVFMP), can be defined as follows. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a complete undirected graph, in which the vertices v_i with $i \in \{1, \ldots, d\}$ represent depots with infinite capacity, while the n vertices v_i with $i \in \{d+1, \ldots, d+n\}$ stand for customers with a demand for a non-negative amount of product q_i. The edges $(i,j) \in \mathcal{E}$ represent the possibility of traveling between vertices v_i and v_j for a total travel distance of c_{ij}. Finally, w types of vehicles are available in unlimited quantity, any vehicle k being characterized by a base acquisition/depreciation cost e_k, a per-distance-unit cost u_k, and a capacity Q_k. The overall fleet, containing a large number of vehicles of each types, is notated $\mathcal{F}$. As such, the minimum cost $\Phi(x,q)$ to perform a route with distance x and total demand q is given by

$$\Phi(x,q) = \min_{k \in \mathcal{F}/q \leqslant Q_k} \{e_k + u_k x\}. \tag{1}$$

The MDVFMP aims to find a set of routes, as well as their assignment to vehicles and depots, to service each customer once and minimize the total cost. Each route assigned to any vehicle k must start and end at the same depot location and carry less than Q_k of units of products. A mathematical formulation is given in Eqs. (2)–(9). The binary variables x_{ijko} take value 1 if and only if vertex v_j is visited immediately after v_i by a vehicle k from depot o. The objective, presented in Eq. (2), includes the base cost of e_k of any used vehicle k, and the distance-based cost $u_k c_{ij}$. Eq. (3) constrains each customer to be visited once. For every depot and vehicle, Eq. (4) limits the number of routes to one, Eq. (5) ensures the consistency between the depot assignment, and the origin location and Eq. (6) ensures the conservation of the flow. Eq. (7) enforces the capacity limits for the vehicles, and sub-tours are eliminated by Eq. (8).

$$\text{Minimize} \sum_{k\in\mathcal{F}} \sum_{o=1}^{d} \left(\sum_{i=d+1}^{d+n} e_k x_{oiko} + \sum_{i=1}^{d+n}\sum_{j=1}^{d+n} u_k c_{ij} x_{ijko} \right) \tag{2}$$

$$\text{s.t.} \sum_{j=1}^{d+n}\sum_{o=1}^{d}\sum_{k\in\mathcal{F}} x_{ijko} = 1 \qquad i = d+1, \ldots, d+n \tag{3}$$

$$\sum_{j=1}^{d+n} x_{ojko} \leqslant 1 \qquad k \in \mathcal{F};\ o = 1, \ldots, d \tag{4}$$

$$\sum_{o' \in \{1,\ldots,d\}-\{o\}} \sum_{j=1}^{d+n} x_{o'jko} = 0 \qquad k \in \mathcal{F};\ o = 1, \ldots, d \tag{5}$$

$$\sum_{j=1}^{d+n} x_{jiko} - \sum_{j=1}^{d+n} x_{ijko} = 0 \qquad i = 1, \ldots, d+n;\ k \in \mathcal{F};\ o = 1, \ldots, d \tag{6}$$

$$\sum_{i=d+1}^{d+n}\sum_{j=1}^{d+n} q_i x_{ijko} \leqslant Q_k \qquad k \in \mathcal{F};\ o = 1, \ldots, d \tag{7}$$

$$\sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{S}} x_{ijko} \leqslant |\mathcal{S}| - 1 \qquad \mathcal{S} \subset \{d+1, \ldots, d+n\}$$
$$|\mathcal{S}| \geqslant 2;\ k \in \mathcal{F};\ o = 1, \ldots, d \tag{8}$$

$$x_{ijko} \in \{0,1\} \qquad i = 1, \ldots, d+n;\ j = 1, \ldots, d+n;$$
$$k \in \mathcal{F};\ o = 1, \ldots, d \tag{9}$$

The MDVFMP includes several prominent problems as special cases, such as the Vehicle Fleet Mix Problem (VFMP) when $d = 1$, the Multi-Depot VRP (MDVRP) when $w = 1$, and the Capacitated VRP (CVRP) when $(w, d) = (1, 1)$. The MDVFMP is also NP-hard as a generalization of the CVRP.

Contributions on the MDVFMP are not frequent in the literature, and we are only aware of two published methods. Salhi and Sari (1997) introduced a neighborhood-based method relying on the same concepts as the Variable Neighborhood Search, changing the neighborhoods by increasing size whenever a local optimum is encountered. Advanced moves that impact both the assignment and the sequencing are used once several simpler neighborhoods have been exhausted. More recently, Salhi, Imran, and Wassan (2013) proposed a variable neighborhood search, which identifies borderline customers and applies route aggregation and disaggregation techniques.

The MDVFMP is also a special case of the problems addressed by Irnich (2000), Dondo and Cerdá (2007), and Goel and Gruhn (2008). Yet, a limited fleet has been considered in most cases along with some other problem attributes, leading to different solution techniques.

In contrast, the literature dedicated to its two immediate sub-problems, the MDVRP and the VFMP, is much more furnished. An extensive survey of all methods for these two sub-problems is outside the scope of this section, and we refer to Ombuki-Berman and Hanshar (2009), Vidal et al. (2013c), and Subramanian, Penna, Uchoa, and Ochi (2012) to that extent. Several meta-heuristics produce solutions of remarkable quality. For the MDVRP, the current state of the art results are produced by the Hybrid Genetic Search with Advanced Diversity Control (HGSADC) of Vidal et al. (2012), which relies on efficient crossover and LS-improvement procedures to create new individuals. Of particular interest is the individual evaluation used in HGSADC, which relies on both solution