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Decision Support

Meta-data envelopment analysis: Finding a direction towards marginal profit maximization

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ABSTRACT

This paper discusses a new meta-DEA approach to solve the problem of choosing direction vectors when estimating the directional distance function. The proposed model emphasizes finding the "direction" for productivity improvement rather than estimating the "score" of efficiency; focusing on "planning" over "evaluation". In fact, the direction towards marginal profit maximization implies a step-by-step improvement and "wait-and-see" decision process, which is more consistent with the practical decision-making process. An empirical study of U.S. coal-fired power plants operating in 2011 validates the proposed model. The results show that the efficiency measure using the proposed direction is consistent with all other indices with the exception of the direction towards the profit-maximized benchmark. We conclude that the marginal profit maximization is a useful guide for determining direction in the directional distance function.

1. Introduction

Data envelopment analysis (DEA) is a nonparametric approach used to estimate the production function and efficiency of each decision-making unit (DMU) in order to support the performance evaluation of production systems (Charnes, Cooper, & Rhodes, 1978). Given an orientation, although DEA can identify the "position" (i.e., efficiency score) of DMU in the production possibility set, it is not clear to justify the "direction" (orientation) we choose for projecting an inefficient firm to the frontier. In particular, it is an unsolved issue to choose the direction vectors when estimating the directional distance function (DDF) (Färe, Grosskopf, & Whittaker, 2013).

Previous studies focus on efficiency estimation, i.e., they estimate efficiency using radial-based or slack-based approach given a specific orientation. Debteu (1951) and Farrell (1957) developed indices of technical efficiency measured as the maximum radial reduction in all inputs consistent with the equivalent production of observed output. However, additional slack may still exist in the use of some, but not all inputs, and as a result, a Farrell efficient producer may not be Koopmans efficient (Koopmans, 1951). Therefore, the Russell measure (Färe & Lovell, 1978), the additive DEA model (Charnes, Cooper, Golany, Seiford, & Stutz, 1985), and the slacks-based measure (Tone, 2001) have been proposed to identify an inefficient firm with additional slack and to support Koopmans efficiency (Koopmans, 1951). In addition, in order to contract input level and expand output level simultaneously, the hyperbolic measure (Färe, Grosskopf, & Lovell, 1985; Färe, Grosskopf, Lovell, & Pasurka, 1989) and the directional distance function (DFF) (Chambers, Chung, & Färe, 1996; Chung, Färe, & Grosskopf, 1997) have been developed, particularly to address the undesirable outputs.[1]

The DEA literature focuses primarily on efficiency estimation rather than researching the direction for productivity improvement. Typically, the choice of input-oriented or output-oriented DEA models will depend upon the production process characterizing a firm (i.e., minimize the use of inputs to produce a given level of output or maximize the level of output given the levels of the inputs). Lovell (1993) suggested the use of input-oriented models in cases when producers are required to meet market demand and can freely adjust the input usage, whereas output-oriented models are helpful in cases when producers cannot freely adjust the input usage, but can build the capacity flexibility that allows changing the output. Theoretically, the private sector, which is not subject to budget and personnel limitations, would use an input-oriented model, whereas the public sector would use an output-oriented model.

Recent studies have focused on finding the proper direction towards the efficient frontier, including the productivity improvement of an efficient firm on the frontier. Frei and Harker (1999)

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[1] The production of desirable output is typically accompanied by the joint production of undesirable by-products, such as pollution. It implies that the reduction of bads will be costly: either abatement capital must be used, production must be cut back, or fines must be paid (Chung et al., 1997).

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determined the least-norm projection from an inefficient firm to the frontier, but this direction is non-proportional and is not unit-invariant. Extending their work, Baek and Lee (2009) proposed a least-distance measure to represent the shortest projection satisfying unit-invariant property. Aparicio, Ruiz, and Sirvent (2007) found the closest targets of each inefficient firm according to the similarity calculated by distance functions and interpreted them as the closeness between the inputs and outputs of the assessed unit and the proposed targets. Zofio, Paster, and Aparicio (2013) suggested choosing the direction in the direction distance function (DDF) to move towards the allocatively efficient point, i.e., "When market prices are observed and firms have a profit maximizing behavior, it seems natural to choose as the directional vector that projecting inefficient firms towards profit maximizing benchmarks." Färe et al. (2013) suggested choosing the endogenous direction vectors based on exogenous normalization constraints when estimating the DDF. However, it is void of economic meanings. Lee and Johnson (2012a), who extended to oligopolistic markets, suggested that each firm should select the direction for improvement in the DDF to move towards its Nash equilibrium benchmark.

This paper suggests a direction towards marginal profit maximization. This new approach is based on three ideas. First, it is difficult in practice to apply the output-oriented model without the ability to adjust the input resources; on the other hand, it is also difficult to decrease input levels without a decrease of output level. Second, when attempting to adjust the input and output levels, the lack of a relationship between inputs and outputs (i.e., marginal productivity, MP) makes it difficult to characterize the moving process. Third, it is preferable to adopt a wait-and-see criterion due to an uncertain environment (Benavides, Duley, & Johnson, 1999; Lee & Chien, 2013) rather than assuming that a firm can move towards profit maximization benchmark in one shot. The wait-and-see can also response to the change of the production function and marginal productivity over time during the moving process (Førsund & Hjalmarsson, 1974; Lee & Johnson, 2014).

Marginal productivity (MP) has an important role in economic theory and applications. The purpose of estimating a production function is to obtain estimates of the regression coefficients. These coefficients refer to the marginal productivities which characterize how the dependent variable will be affected by changing one extra unit of independent variables. Economists use the word "elasticity" to measure the percentage of how changing one variable affects the other variables. From an engineering perspective, the estimation of MP also contributes to capacity planning and resource allocation. Capacity is the maximal output level of a production process. The output is a result of the total productive capability of a firm's resources including workforce, machinery, and utilities (Lee & Johnson, 2012b). Capacity adjustment is the ability to adjust the output levels in response to a wide variety of uncertainty, in particular demand fluctuation, by controlling variable resources in the short run (Fine & Freund, 1990). In production theory, capacity adjustment can be interpreted as the MP of the production function, i.e., the extra output generated by one more unit of an input.

Generally, we calculate the MP of the production function by the partial derivatives of the production function given a smooth efficient frontier if we have a functional form. In practice, the production function cannot be observed easily, and therefore the piece-wise linear production function is estimated using DEA based on collected observations (Banker, Charnes, & Cooper, 1984; Charnes et al., 1978; Fried, Lovell, & Schmidt, 2008). However, a piece-wise linear frontier forms a polyhedral set representing production technologies and thus is not differentiable. Podinovski and Førsund (2010) gave an explicit definition of differential characteristics on a non-differentiable efficient frontier and proposed a directional-derivative approach to calculate elasticity measures without any simplifying assumptions. They applied differential characteristics to the DEA frontier and addressed elasticity measures and marginal rates of substitution. This study uses DDF to develop a directional marginal productivity (DMP) which supports an expectable tradeoff between multiple outputs and identifies the marginal profit maximization given price information.

This study both fills the existing gap in the literature as well as contributing to practical capacity planning. We believe that our most significant contribution to the area of productivity research is the stress on "productivity improvement" rather than "efficiency measure", i.e., finding the improving direction. We construct a model to search the best direction for marginal profit maximization, also termed meta-DEA. In fact, to find a direction for marginal profit maximization is more common in practice, for example, a firm may adjust the input resources to control output level (i.e. MP) for addressing demand fluctuation (Lee & Johnson, 2014). In addition, we claim that the direction in which firms usually want to move is to increase both inputs and outputs (e.g., MP) because no company is willing to take the risk of losing sales or market share by contracting its inputs. Technically, of course, this is not strictly an efficiency improvement, but nonetheless, our proposed model is rather a behavioral models than efficiency models and attempts to identify the direction a firm would really change towards rather than to measure efficiency in that direction.

The remainder of this paper is organized as follows. Section 2 introduces the estimation of directional marginal productivity. Section 3 presents our meta-DEA approach and suggests the direction towards marginal profit maximization. Section 4 describes a DDF efficiency measure for both desirable outputs and undesirable outputs. We also compare five approaches using different direction vectors in DDF. Section 5 explains our empirical study of U.S. coal-fired power plants operating in 2011. Section 6 concludes.

2. Directional marginal productivity

As mentioned, marginal productivity (MP) represents the extra output generated by one more unit of an input. There are two common methods to estimate MP. *Stochastic Frontier Analysis* (SFA) estimates the production function with a given functional form, e.g., a simple case of a linear function estimated by ordinary least squares (OLS). The coefficients associated with the independent factors provide estimates of the MP. *Data envelopment analysis* (DEA) constructs a piece-wise linear production function approximating the true production function, and the dual multiplier linear program to the primal envelopment model represents the MP.

Both of these methods have some drawbacks. SFA requires defining a functional form and risks potential misspecification, whereas DEA's observations on the production frontier do not have unique shadow prices, i.e., it is common to have shadow price values of zero. To overcome the problem of nondifferentiability, Podinovski and Førsund (2010) introduced right-hand and left-hand elasticities, which can be viewed as directional derivatives of the optimal value function in a modified envelopment DEA model, and which can be calculated using a variant of the theorem on marginal values in linear programming (Shapiro, 1979). The regular elasticity measure is obtained when the two one-sided elasticities are equal.

This section describes a proposed directional marginal productivity (DMP) model via the directional distance function (DDF). Keeping our model as general as possible, we consider the desirable outputs and undesirable outputs. Recall that the DEA estimator assumes that the good outputs are freely disposable; however, this property cannot be directly applied to bad outputs. Intuitively,