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Decision Support

Variational approach for a general financial equilibrium problem: The Deficit Formula, the Balance Law and the Liability Formula. A path to the economy recovery

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ABSTRACT

Without using a technical language, but using the universal language of mathematics, we provide simple but significant laws, as Deficit Formula, Balance Law and Liability Formula, for the management of the world economy. Decisions, under these laws, for the recovery of the economy and for the good governance clearly appear. Further a simple but useful economical indicator $E(t)$ is provided and the results are illustrated with a significant example.

1. Introduction

In this paper a general equilibrium model of financial flows and prices is considered. The model is assumed to evolve in time. The equilibrium conditions are considered in a dynamic sense and the governing variational inequality formulation is presented. Using delicate tools of Nonlinear Analysis and especially some properties of the dual model (the so-called *shadow* problem), which are not always intuitive, the optimal composition of assets, liabilities and prices is determined and a qualitative analysis of the equilibrium is performed. In fact, in order to obtain the recovery of the economy, in Section 7 the propositions which derive from the analysis of the financial equilibrium, namely from the Deficit Formula, the Balance Law and the Liability Formula, are provided for the convenience of the decision-makers of the world economy. These propositions are written not by means of a technical language, but by means of the universal language of mathematics, which allows everyone to evaluate the consequences of each choice in terms of equilibrium. Moreover a simple but useful indicator of the economy $E(t)$ is provided and its effectiveness is illustrated by means of a significant example. The other sections of the paper are organized in the following way: in Section 2 the detailed financial model is presented, in Section 3 we recall the infinite dimensional Lagrange duality, in Section 4 we provide the proofs of the steps which lead to the variational inequality of the governing equilibrium conditions, in Section 5 we give some existence results, in Section 6 we provide an application of the infinite dimensional duality to the general financial equilibrium problem from which the Deficit Formula, the Balance Law and the Liability Formula follow. In Section 8 a significant example illustrates the results obtained and the impact that the components of the model have on the equilibrium.

2. The model

The first authors to develop a multi-sector, multi-instrument financial equilibrium model using the variational inequality theory were Nagurney, Dong, and Hughes (1992). These results were, subsequently, extended by Nagurney (1994, 2001) to include more general utility functions and by Nagurney and Siokos (1997a, 1997b) to the international domain (see also Jaillet, Lamberton, & Lapeyre, 1990; Tourin & Zariphopoulou, 1994 for related papers). In Dong, Zhang, and Nagurney (1996), the authors apply for the

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first time the methodology of projected dynamical systems to develop a multi-sector, multi-instrument financial model, whose set of stationary points coincides with the set of solutions to the variational inequality model developed in Nagurney (1994), and then to study it qualitatively, providing stability analysis results. Recently in Barbagallo, Daniele, and Maugeri (2012); Daniele (2003a, 2003b, 2005); Daniele, Giuffrè, and Pia (2005) more general models have been studied allowing that the data are evolving over time. The more significant paper in this sense is Barbagallo et al. (2012), which we generalize and in addition we enrich by introducing Section 7 devoted to the economy recovery.

Now we describe in detail the model we are dealing with. We consider a financial economy consisting of m sectors, for example households, domestic business, banks and other financial institutions, as well as state and local governments, with a typical sector denoted by i, and of n instruments, for example mortgages, mutual funds, saving deposits, money market funds, with a typical financial instrument denoted by j, in the time interval $[0,T]$. Let $s_i(t)$ denote the total financial volume held by sector i at time t as assets, and let $l_i(t)$ be the total financial volume held by sector i at time t as liabilities. Then, unlike previous papers (see Daniele, 2003a, 2003b, 2005, 2006, 2010; Daniele et al., 2005), we allow markets of assets and liabilities to have different investments $s_i(t)$ and $l_i(t)$, respectively. Since we are working in the presence of uncertainty and of risk perspectives, the volumes $s_i(t)$ and $l_i(t)$ held by each sector cannot be considered stable with respect to time and may decrease or increase. For example, depending on the crisis periods, a sector may decide not to invest on instruments and to buy goods as gold and silver. At time t, we denote the amount of instrument j held as an asset in sector i's portfolio by $x_{ij}(t)$ and the amount of instrument j held as a liability in sector i's portfolio by $y_{ij}(t)$. The assets and liabilities in all the sectors are grouped into the matrices

$$x(t)=\begin{bmatrix} x_1(t)\\ \dots\\ x_i(t)\\ \dots\\ x_n(t)\end{bmatrix}=\begin{bmatrix} x_{11}(t) & \dots & x_{1j}(t) & \dots & x_{1n}(t)\\ \dots & \dots & \dots & \dots & \dots\\ x_{i1}(t) & \dots & x_{ij}(t) & \dots & x_{in}(t)\\ \dots & \dots & \dots & \dots & \dots\\ x_{m1}(t) & \dots & x_{mj}(t) & \dots & x_{mn}(t)\end{bmatrix}$$

and

$$y(t)=\begin{bmatrix} y_1(t)\\ \dots\\ y_i(t)\\ \dots\\ y_n(t)\end{bmatrix}=\begin{bmatrix} y_{11}(t) & \dots & y_{1j}(t) & \dots & y_{1n}(t)\\ \dots & \dots & \dots & \dots & \dots\\ y_{i1}(t) & \dots & y_{ij}(t) & \dots & y_{in}(t)\\ \dots & \dots & \dots & \dots & \dots\\ y_{m1}(t) & \dots & y_{mj}(t) & \dots & y_{mn}(t)\end{bmatrix}.$$

We denote the price of instrument j held as an asset at time t by $r_j(t)$ and the price of instrument j held as a liability at time t by $(1+h_j(t))r_j(t)$, where h_j is a nonnegative function defined into $[0,T]$ and belonging to $L^\infty([0,T],\mathbb{R})=\{f:[0,T]\to\mathbb{R}$ measurable : $\exists c\geqslant 0$ such that $|f(t)|\leqslant c$, a.e. in $[0,T]\}$ endowed with the norm $\|f(t)\|_{L^\infty([0,T],\mathbb{R})}=\inf\{c\geqslant 0:\ |f(t)|\leqslant c,$ a.e. in $[0,T]\}$. We introduce the term $h_j(t)$ because the prices of liabilities are generally greater than or equal to the prices of assets in order to describe, in a more realistic way, the behavior of the markets for which the liabilities are more expensive than the assets. In such a way, this paper appears as an improvement in various directions of the previous ones (Daniele, 2003a, 2003b, 2005, 2006, 2010 & Daniele et al., 2005). We group the instrument prices held as an asset into the vector $r(t)=[r_1(t),r_2(t),\dots,r_i(t),\dots,r_n(t)]^T$ and the instrument prices held as a liability into the vector $(1+h(t))r(t)=[(1+h_1(t))r_1(t),(1+h_2(t))r_2(t),\dots,(1+h_i(t))r_i(t),\dots,(1+h_n(t))r_n(t)]^T$. In our problem the prices of each instrument appear as unknown variables. Under the assumption of perfect competition, each sector will behave as if it has no influence on the instrument prices or on the behavior of the other sectors, but on the total amount of the investments and the liabilities of each sector.

In order to express the time dependent equilibrium conditions by means of an evolutionary variational inequality, we choose as a functional setting the very general Lebesgue space $L^2([0,T],\mathbb{R}^p)=\left\{f:[0,T]\to\mathbb{R}^p \text{ measurable}:\ \int_0^T\|f(t)\|_p^2dt<+\infty\right\}$, with the norm $\|f\|_{L^2([0,T],\mathbb{R}^p)}=\left(\int_0^T\|f(t)\|_p^2dt\right)^{\frac{1}{2}}$. Then, the set of feasible assets and liabilities for each sector $i=1,\dots,m$, becomes

$$P_i=\Bigg\{(x_i(t),y_i(t))\in L^2([0,T],\mathbb{R}^{2n}):\ \sum_{j=1}^n x_{ij}(t)=s_i(t),\quad \sum_{j=1}^n y_{ij}(t)=l_i(t)\text{ a.e. in }[0,T],\quad x_i(t)\geqslant 0,\ y_i(t)\geqslant 0,\ \text{a.e. in }[0,T]\Bigg\}\quad \forall i=1,\dots,m.$$

In such a way the set of all feasible assets and liabilities becomes

$$P=\Bigg\{(x(t),y(t))\in L^2([0,T],\mathbb{R}^{2mn}):\quad \sum_{j=1}^n x_{ij}(t)=s_i(t),\quad \sum_{j=1}^n y_{ij}(t)=l_i(t),\forall\ i=1,\dots,m,\ \text{a.e. in }[0,T],\quad x_i(t)\geqslant 0,\quad y_i(t)\geqslant 0,\ \forall\ i=1,\dots,m,\ \text{a.e. in }[0,T]\Bigg\}.$$

Now, in order to improve the model of competitive financial equilibrium described in Barbagallo et al. (2012), which represents a significant but still partial approach to the complex problem of financial equilibrium, we consider the possibility of policy interventions in the financial equilibrium conditions and incorporate them in form of taxes and price controls and, mainly, we consider a more complete definition of equilibrium prices $r(t)$, based on the demand–supply law, imposing that the equilibrium prices vary between floor and ceiling prices and in Section 7 we provide useful suggestions for the recovery of the economy.

To this aim, denote the ceiling price associated with instrument j by $\bar{r}_j$ and the nonnegative floor price associated with instrument j by $\underline{r}_j$, with $\bar{r}_j(t)>\underline{r}_j(t)$, a.e. in $[0,T]$. The floor price $\underline{r}_j(t)$ is determined on the basis of the official interest rate fixed by the central banks, which in turns take into account the consumer price inflation. Then the equilibrium prices $r_j^*(t)$ cannot be less than these floor prices. The ceiling price $\bar{r}_j(t)$ derives from the financial need to control the national debt arising from the amount of public bonds and of the rise in inflation. It is a sign of the difficulty on the recovery of the economy. However it should be not overestimated because it produces an availability of money.

In detail, the meaning of the lower and upper bounds is that to each investor a minimal price $\underline{r}_j$ for the assets held in the instrument j is guaranteed, whereas each investor is requested to pay for the liabilities in any case a minimal price $(1+h_j)\underline{r}_j$. Analogously each investor cannot obtain for an asset a price greater than $\bar{r}_j$ and as a liability the price cannot exceed the maximum price $(1+h_j)\bar{r}_j$.

Denote the given tax rate levied on sector i's net yield on financial instrument j, as τ_{ij}. Assume that the tax rates lie in the interval $[0,1)$ and belong to $L^\infty([0,T],\mathbb{R})$. Therefore, the government in this model has the flexibility of levying a distinct tax rate across both sectors and instruments.

Let us group the instrument ceiling prices $\bar{r}_j$ into the column vector $\bar{r}_j(t)=[\bar{r}_1(t),\dots,\bar{r}_i(t),\dots,\bar{r}_n(t)]^T$, the instrument floor prices