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Decision Support

Downturn Loss Given Default: Mixture distribution estimation

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ABSTRACT

The internal estimates of Loss Given Default (LGD) must reflect economic downturn conditions, thus estimating the "downturn LGD", as the new Basel Capital Accord Basel II establishes. We suggest a methodology to estimate the downturn LGD distribution to overcome the arbitrariness of the methods suggested by Basel II. We assume that LGD is a mixture of an expansion and recession distribution. In this work, we propose an accurate parametric model for LGD and we estimate its parameters by the EM algorithm. Finally, we apply the proposed model to empirical data on Italian bank loans.

1. Introduction

Loss Given Default (LGD) is the loss incurred by a financial institution when an obligor defaults on a loan, given as the fraction of Exposure At Default (EAD) unpaid after some period of time. In the Basel II framework (Basel Committee on Banking Supervision, 2004, paragraph 286–317), banks adopting the advanced Internal Rating Based (IRB) approach are allowed to use their own estimates of LGDs that have to reflect economic downturn conditions. Hence, the "downturn LGD" is the maximum of the long-run default-weighted average LGD and the stressed LGD.

It requires the banks to identify the appropriate downturn conditions and incorporate them so as to produce LGD parameters for the bank's exposures, which are consistent with the identified downturn conditions. The main reason for this requirement is that the Vasicek model (Vasicek, 2002) used in Basel II does not have systematic correlation between Probability of Default (PD) and LGD and, to compensate for this deficiency, downturn LGD are required to be used as input to the model.

Although the downturn LGD is a key variable for banking practice, such a pivotal topic is relatively unexplored in the literature. The main aim of this paper is to propose a methodology to estimate the downturn LGD distribution. To achieve this aim, we consider the dynamic behaviour of LGD over the economic cycle characterised by two regimes: expansion and recession.

We assume that the LGD is a mixture of an expansion and a recession distributions, each of these distributions is given by the mixture of a Bernoulli random variable and a beta random variable, as Calabrese (in press) suggested. On the one hand, the Bernoulli random variable allows to reproduce the high concentration of data at total recovery and total loss (Calabrese & Zenga, 2010; Renault & Scaillet, 2004; Schuermann, 2003). On the other hand, the beta distribution is well suited[1] to the modelling of LGDs (Bruche & González-Aguado, 2008; Gupton, Finger, & Bhatia, 1997; Gupton & Stein, 2002). To estimate the parameters of the downturn LGD distribution, we apply the EM algorithm (Dempster, Laird, & Rubin, 1977). To obtain a finite beta density function, Calabrese and Zenga (2010)'s parametrization is used. With this method banks do not need to identify arbitrarily downturn conditions and, unlike the factor method, data on the default risk and default correlation are not required. Finally, we apply this proposal to a comprehensive Bank of Italy data set (Bank of Italy, 2001) of 149,378 Italian bank loans and we compare it with some methods used in the literature to estimate the downturn LGD.

The present paper is organised as follows. The next section analyses the available literature on downturn LGD. Section 3 describes some approaches to estimate downturn LGD. The following section presents the proposed approach to estimate the downturn LGD distribution. Section 5 describes the dataset of the Bank of Italy and shows the estimation results by applying the proposed model to these data. Finally, the last section is devoted to conclusions.

2. Literature review

An extensive literature suggested a link between LGD and the economic cycle (e.g. Bellotti & Crook, 2012; Calabrese, in press).

[1] Since LGD lies in the interval [0,1], the beta distribution is a suitable parametric model for LGDs since it has support [0,1] and, in spite of requiring only two parameters, is quite flexible.

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The systematic correlation between PD and LGD is not taken into account in many models. In the standard rating-based credit risk model developed by Gupton et al. (1997), it is assumed that recoveries on defaulted exposures are random outcomes, independent of the default event. A similar independence assumption is made in the model of Jarrow, Lando, and Turnbull (1997) and in the Vasicek model (Vasicek, 2002) used in the Basel II Accord. However, if realizations of recoveries are low exactly at times when many firms default, the assumption that recoveries are independent of default rates or constant would result in an underestimation of credit risk. To compensate for this deficiency, downturn LGD estimates are required to be used as an input to the model.

In the Basel II Accord (BCBS, 2005b), two approaches are presented to estimate downturn LGD. One approach would be to apply a mapping function similar to that used for the PDs that would extrapolate downturn LGDs from bank-reported average LGDs. Alternatively, banks could be asked to provide downturn LGD figures based on their internal assessments LGDs during adverse conditions. Provided that data is available, the latter approach is the easiest to implement, so Basel III (BCBS, 2011, paragraph 20) considers only this method to compute downturn LGD.

The drawback is that LGD data is generally sparse and there is very limited industry experience with regard to LGD estimates. Downturn LGD estimation based on historical data is currently not possible for many banks because of the short time periods available or for the lack of an economic downturn during the available period. The first approach of Basel II is an appropriate solution when historical data is not available.

Following the first approach of Basel II, Miu and Ozdemir (2006) suggest that the original LGD assessment by banks, without considering PD and LGD correlation, can be appropriately adjusted by incorporating a certain degree of conservatism in cyclical LGD estimates within a point-in-time modelling framework. They use Monte Carlo to tabulate the relationship between long-run and downturn LGD. Barco (2007) extends their work to develop an analytical relationship between long-run and downturn LGD.

Moreover, Sabato and Schmid (2008) suggest a simple mapping function to estimate downturn LGD. They investigate the relationship between LGD and the credit cycle over the period from 2002 to 2007 using data covering a set of retail loans. The linear mapping function proposed by the Board of Governors of the Federal Reserve Bank (2006) can be considered as a particular case of Sabato and Schmid's proposal, as explained in Section 3.

Both the approaches presented by Basel II show the pivotal drawback of how economic downturn or mapping function of average LGD should be defined and identified. This arbitrariness leads to very different approaches being implemented across banks and countries, and significant effects on the level of capital requirements. As many authors have shown (e.g., Saurina & Trucharte, 2007; Altman & Sabato, 2005), Basel II Advanced-IRB capital requirements are highly sensible to LGD values in particular for retail asset classes. Hence, there is the necessity of suggesting a method to estimate downturn LGD to overcome the arbitrariness of the approaches suggested by Basel II.

3. Downturn LGD estimation models

Following the first approach of Basel II (BCBS, 2005b), Sabato and Schmid (2008) suggest the following linear mapping function to estimate downturn LGD on unsecured positions

$$DLGD - \mu^{LGD} = LGDSF(1 - \mu^{LGD}) \quad (1)$$

where μ^{LGD} is the long-term average[2] LGD, DLGD is the expected Downturn LGD and LGDSF is the LGD Stressing Factor given by

$$LGDSF = \frac{\text{stressed}LGD - LGD}{LGD}. \quad (2)$$

Sabato and Schmid (2008) suggest to compute the stressed LGD as a function (not specified) of the stressed PD, given by the average PD plus the standard deviation of the observed default rates. The mapping function (1) implies that debts with relatively low historical LGD rates (e.g. senior bank loans) should have relatively large adjustments to their long-term average LGD rates, while debts with high historical LGD rates (e.g. subordinated bonds) should have relatively small adjustments. In other words, the difference between downturn LGD and the long-term average LGD varies inversely with the long-term average LGD.

In recognition that banks may be unable to estimate the LGD stressing factor, the Board of Governors of the Federal Reserve Bank (2006) proposes the following particular case of Eq. (1) with $LGDSF = 0.08$

$$DLGD = .08 + .92\mu^{LGD}. \quad (3)$$

where μ^{LGD} equals the long-term average LGD and DLGD is the expected Downturn LGD. It is worth noting that the magnitudes of the proposed adjustments to LGD are relatively modest, with a maximum adjustment of only eight percentage points. The Federal Reserve has offered no justification for the linear mapping function (3) except perhaps its intuitive appeal that debts with the lowest historical average LGD rates receive the largest upward downturn LGD adjustments.

The methodology suggested by Miu and Ozdemir (2006) and then generalised by Barco (2007) also belongs to the first approach of Basel II. To analyse Barco's model, we start with the well-known Merton framework where the log return of the obligor i's asset value is given by

$$A_i = \rho_i^{PD} X_m + \sqrt{1 - (\rho_i^{PD})^2} X_{is}$$

where ρ_i^{PD} is known as correlation of asset returns. The independent standard normal random variables X_m and X_{is} are the systematic factor and obligor-specific idiosyncratic factor, respectively. Barco (2007) models the value of the asset of the creditor with a lognormal distribution. This is achieved by first establishing the following relationship for the standardised asset return

$$R_i = \rho_i^R X_m + \sqrt{1 - (\rho_i^R)^2} Y_m$$

where Y_m is an independent standard normal random variable representing the residual systematic asset return not explained by X_m. The parameter ρ_i^R correlates the assets of the obligor to the systemic factor X_m. Assuming obligor exposure is one unit with mean default rate denoted by μ_i^{PD}, its loss random variable is defined by

$$L_i = 1_{\{A_i < \Phi^{-1}(\mu_i^{PD})\}} L_i^R$$

where $1_{\{\}}(\cdot)$ is the indicator function and $\Phi^{-1}(\cdot)$ is the quantile function of the standard normal distribution.

Assuming a fully granular portfolio, L_i is contained in a homogeneous portfolio and continue to suppose that its exposure is equal to one unit, so Barco (2007) defines

$$L^{\infty} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} L_i.$$

[2] μ^{LGD} is known as *long-run default-weighted average loss rate given default* in Basel II (BCBS, 2005a).