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A multistage linear stochastic programming model for optimal corporate debt management

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ABSTRACT

Large corporations fund their capital and operational expenses by issuing bonds with a variety of indexations, denominations, maturities and amortization schedules. We propose a multistage linear stochastic programming model that optimizes bond issuance by minimizing the mean funding cost while keeping leverage under control and insolvency risk at an acceptable level. The funding requirements are determined by a fixed investment schedule with uncertain cash flows. Candidate bonds are described in a detailed and realistic manner. A specific scenario tree structure guarantees computational tractability even for long horizon problems. Based on a simplified example, we present a sensitivity analysis of the first stage solution and the stochastic efficient frontier of the mean-risk trade-off. A realistic exercise stresses the importance of controlling leverage. Based on the proposed model, a financial planning tool has been implemented and deployed for Brazilian oil company Petrobras.

1. Introduction

In large corporations, the goal of debt management is the dynamic bond issuance under uncertainty, with the purpose of optimally funding their capital and operational expenses. Debt portfolios are structured as a mix of securities with differing indexations, denominations, maturities and amortization schedules, in an attempt to balance the expected cost of servicing the debt with risks inherent to interest rates, corporate revenues and costs. In addition to corporate and regulatory operational constraints, debt management must take into account fluctuations in total debt, assets and cash savings, along with other financial performance measures affecting the company's stock price and credit rating. In face of the required modeling flexibility, there is a firmly established literature with applications of *Multistage Stochastic Programming* (MSP) techniques to debt management and, more generally, to *Asset Liability Management* (ALM) problems. Starting with Bradley and Crane (1972), ALM models have been developed for several different applications including insurance companies Cariño and Ziemba (1998) and pension funds Kouwenberg (2001), Hilli, Koivu, Pennanen, and Ranne (2007), and Chiu and Wong (2012). More recently, similar techniques were especialized for optimal sovereign bond issuance, also dealing with the trade-off between minimum expected cost and minimum risk Balibek and Köksalan (2010), Consiglio and Staino (2010), and Date, Canepa, and Abdel-Jawad (2011). For the corporate case, Xu and Birge (2006) introduce a simplified model that maximizes shareholder value over production strategy and dividend distribution policy, considering a single short term debt instrument. However, their model requires the availability of known risk neutral probabilities, an unrealistic assumption especially for companies without a portfolio of tradable assets. To the best of our knowledge, the literature lacks models describing corporate bond issuance under uncertainty, dealing with both the complexity of the dynamic decision process and the trade-off among expected costs, risks and financial performance measures, as observed in practice.

In this article, we present an MSP model for a corporation financing a predetermined set of projects, considering a universe of fixed and floating rates debt instruments. Uncertainty is represented by an event tree with a hybrid information structure, used to avoid exponential complexity with the number of stages. In the first part of the horizon, we build a detailed event tree with a full range of debt instruments available to the decision maker. For the other portion of the time horizon, the event tree is formed by a subsample approximation of uncertainty realizations, with a predetermined policy rule allowing only short-term debt. Our optimization model describes the dynamic decision process where, at every yearly stage, the state of the system is represented by the current cash holdings and the past debt portfolio. It takes into

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account the mean-risk trade-off between expected cost of debt service and expected value of corporation insolvency. Additional operational constraints express corporate debt valuation and the current asset value used to compute the leverage ratio at each stage. Lewellen and Emery (1986) asserts that most reasonable characterizations of corporate debt management policies adopt a borrowing strategy organized around leverage ratio targets. We integrate this performance measure into the objective function, modeling it as a convex piecewise linear penalty of the computed excess leverage.

For an illustrative example with null cash returns and no intermediate penalties, we present a sensitivity analysis of the risk aversion level. Considering different scenario trees, we solve the problem for each risk aversion level and compute efficient frontiers and related solutions. For a realistic example with stochastic cash returns, we make a sensitivity analysis of the excess leverage penalties and show the importance of our multi-criteria objective function to obtain suitable policies. Computations were carried out with a financial planning software tool implemented for a financial and risk management group at Brazilian oil company Petrobras. In our illustration we consider a fictitious, although realistic, project data set.

The remaining content of this article is organized as follows. Section 2 describes our multistage stochastic programming model, with a comprehensive presentation of all elements in the formulation. In Section 3, we perform a series of sensitivity analyses of the optimal solution considering an illustrative example. Section 4 we present the assumptions of a realistic application of our model to the oil industry and show the importance of the excess leverage penalties. Finally, Section 5 summarizes the contributions of this paper and outlines the directions of future research.

2. Multistage stochastic programming model

Multistage stochastic programming is a natural framework for long-term financial planning problems, corporate debt management in particular. The model must describe a dynamic setting where, at a given stage, a decision is taken facing an unknown future. Once decisions are implemented, the next period information is revealed and the process is repeated for the next stage.

A standard approach in MSP models is to represent uncertainty by a discrete event tree, where nodes indicate the state of the process at decision points and arcs the realizations of uncertainty before the next stage. Formally, the information structure given by an event tree can be understood as a filtered probability space Consiglio and Staino (2010) generating a deterministic equivalent of the MSP model. A complete path in the event tree is called a *scenario* and a policy is defined as the set of decisions for all stages and scenarios. This information structure requires that decisions be based solely on past information, expressed in the MSP model formulation by the *non-anticipativity* constraints, which stipulate decision variables at a given stage must be equal if their scenarios share the same node in the event tree.

Given this tree structure, we can immediately observe that the size of the deterministic equivalent grows exponentially with the number of stages. Some authors have dealt with this *curse of dimensionality* applying large scale optimization techniques Pereira and Pinto (1991), Rockafellar and Wets (1991), Shapiro, Tekaya, da Costa, and Soares (2013), while others approximate the original multistage problem by reducing the number decision variables with the adopting single policy rule Rush, Mulvey, Mitchell, and Willemain (2000). A policy rule is a function of the uncertainty realization that generates a unique sequence of feasible decisions for each time of the planning horizon. This framework fits into the independent scenario structure as stated in Rush et al. (2000), however it usually leads to a suboptimal solution when compared to the original multistage one. Indeed, one could define a set of policy rules generally leading to a non-convex optimization problem.

With the purpose of reducing the high dimensionality of our final formulation, we propose a hybrid approach comprising a traditional multistage model for the first T^* periods and an independent-scenario structure with simple fixed-policy rule for $t > T^*$. For the latter, we represent uncertainty by independent scenarios generated from a subsample of the full event tree. In our model, a full set of securities is considered for $t \leqslant T^*$, while for $t > T^*$ we allow only short term bonds to ensure the minimum cash threshold. This structure is motivated by the observation that, in practice, most investments take place at the early stages of the planning horizon where the decision process is described in more detail.

2.1. Definitions

Preparing a complete formal statement of the model, let us first define parameters, risk factors and decision variables used in the formulation. Observe that, by definition, the short term bond type is the first element of the set of fixed rate bonds $\mathcal{X}$.

Scalar parameters

T: Planning horizon
T^*: Detailed planning horizon
S: Number of scenarios
ω: Weighted average cost of capital
c: Initial cash
p: Risk aversion parameter
nX: Number of fixed rate bonds
nY: Number of floating rate bonds
K: Number of leverage targets
$\bar{\psi}$: Average risk premium for pre-existing floating rate bonds
$\bar{\rho}$: Risk free interest and cash account return rate for the time period preceding the planning horizon

Sets

$\mathcal{H} = \{0, \ldots, T-1\}$
$\mathcal{H}^* = \{0, \ldots, T^*-1\}$
$\mathcal{S} = \{1, \ldots, S\}$
$\mathcal{K} = \{1, \ldots, K\}$
$\mathcal{X} = \{1, \ldots, nX\}$
$\mathcal{Y} = \{1, \ldots, nY\}$

Vector parameters

γ_k: Leverage ratio for target $k \in \mathcal{K}$
θ_k: Penalty for excess leverage exceeding target $k \in \mathcal{K}$
x_t: Payment at $t \in \mathcal{H} \cup \{T\}$ of pre-existing fixed-rate bonds
y_t: Outstanding face value at $t \in \mathcal{H} \cup \{T\}$ of pre-existing floating rate bonds
Δy_t: Amortization at $t \in \mathcal{H} \cup \{T\}$ of pre-existing floating rate bonds
M_X^i: Maturity of fixed rate bond $i \in \mathcal{X}$ defined when $M_X^i \leqslant T - T^* + 1$, with $M_X^1 = 1$ corresponding to a short term bond
M_Y^i: Maturity of floating rate bond $i \in \mathcal{Y}$, defined when $M_Y^i \leqslant T - T^* + 1$
ΔX_j^i: Amortization schedule of fixed rate bond $i \in \mathcal{X}$, for payment $j \in \{1, \ldots, M_X^i\}$, where $\sum_{j=1}^{M_X^i} \Delta X_j^i = 1$
ΔY_j^i: Amortization schedule of floating rate bond $i \in \mathcal{Y}, j \in \{1, \ldots, M_Y^i\}$, where $\sum_{j=1}^{M_Y^i} \Delta Y_j^i = 1$