European Journal of Operational Research 237 (2014) 323–334

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Minimizing shifts for personnel task scheduling problems: A three-phase algorithm

Shih-Wei Lin [a,b,1], Kuo-Ching Ying [c,*,1]

[a] Department of Information Management, Chang Gung University, Taoyuan, Taiwan
[b] Department of Medical Research and Development Linkou Branch, Chang Gung Memory Hospital, Linkou, Taiwan
[c] Department of Industrial Engineering and Management, National Taipei University of Technology, Taipei, Taiwan

ARTICLE INFO

Article history:
Received 8 February 2013
Accepted 19 January 2014
Available online 24 January 2014

Keywords:
Scheduling
Shift minimization
Personnel task scheduling problem

ABSTRACT

The personnel task scheduling problem is a subject of commercial interest which has been investigated since the 1950s. This paper proposes an effective and efficient three-phase algorithm for solving the *shift minimization personnel task scheduling problem* (SMPTSP). To illustrate the increased efficacy of the proposed algorithm over an existing algorithm, computational experiments are performed on a test problem set with characteristics motivated by employee scheduling applications. Experimental results show that the proposed algorithm outperforms the existing algorithm in terms of providing optimal solutions, improving upon most of the best-known solutions and revealing high-quality feasible solutions for those unsolved test instances in the literature.

1. Introduction

Since the pioneering studies of Edie (1954) and Dantzig (1954), the *personnel scheduling problem* (PSP) has continued to be a topic of interest to researchers and practitioners. The PSP consists of the various decisions which must be made, such as the assignment of tasks and/or shift sequences. On the basis of the planning horizon, current PSPs can be divided into three typical classifications (Baker, 1976): *shift scheduling*, *days off scheduling* and *tour scheduling*. In the shift scheduling problem, a roster across a time-of-day planning horizon is specified to meet the staff requirements regarding each shift; in the days off scheduling problem, a roster is set under the constraint that the length of the operating week in the facility does not match the length of an employee's working week; and in the tour scheduling problem, which combines the first two types, both the hours of the day and the days of the week in which each employee must work are determined (Van den Bergh, Beliën, De Bruecker, Demeulemeester, & De Boeck, 2013). The three types of PSPs are common in many application areas, such as service systems, manufacturing systems, transportation systems, call centers, airlines, hotels, hospitals and health care systems.

The ongoing interest of researchers has been motivated by the opportunities for the widespread applications of PSPs. The literature on the PSP exhibits a wide range of research methodologies which can be classified either as exact methods or as those based on heuristic algorithms. Exact methods, which have appeared in PSP research, include: linear programming (Fowler, Wirojanagud, & Gel, 2008; Hochbaum & Levin, 2006; Hojati & Patil, 2011); constraint programming (Laporte & Pesant, 2004; Qu & He, 2009); goal programming (Azaiez & Al Sharif, 2005; Lin, Chen, Chou, & Liao, 2012; Topaloglu & Ozkarahan, 2004), integer programming (Eiselt & Marianov, 2008; Eitzen, Panton, & Mills, 2004; Seckiner, Gokcen, & Kurt, 2007); mixed integer programming (Firat & Hurkens, 2012; Hertz, Lahrichi, & Widmer, 2010; Yilmaz, 2012); column generation (Al-Yakoob & Sherali, 2008; He & Qu, 2012; Restrepo, Lozano, & Medaglia, 2012); dynamic programming (Beliën & Demeulemeester, 2007; Elshafei & Alfares, 2008); and Lagrange relaxation (Bard & Purnomo, 2007; Pot, Bhulai, & Koole, 2008). Although hundreds of exact methods have been proposed in the literature, the computational requirements for obtaining optimal solutions by exact methods are high, even for a moderate-sized PSP. For large-scale PSPs, exact methods may not quickly produce any feasible solutions.

The inherent difficulties of PSPs have encouraged the development of heuristic algorithms for large-scale problems, as required in typical real-world systems, in order to obtain the best solutions within a reasonable computing time (Ernst, Jiang, Krishnamoorthy, & Sier, 2004). The heuristic algorithms in the literature are of two types: *constructive heuristics* and *improvement heuristics*. The constructive heuristic is essentially a single pass method which uses a specific rule to assign a priority index to each employee and, in each step, to specify a shift or task. Once an employee's shift is

* Corresponding author. Tel.: +886 2 2771 2171; fax: +886 2 2731 7168.
E-mail address: kcying@ntut.edu.tw (K.-C. Ying).

[1] These authors contributed equally to this work.

http://dx.doi.org/10.1016/j.ejor.2014.01.035
0377-2217/

determined, it is fixed and cannot be reversed (Ying & Liao, 2003). During the last decade, constructive heuristic algorithms have been proposed for a variety of applications with respect to PSPs, such as nurse scheduling problems (Brucker, Burke, Curtois, Qu, & Vanden Berghe, 2010; Wright, Bretthauer, & Cote, 2006), crew scheduling problems (Elizondo, Parada, Pradenas, & Artigues, 2010; Lin et al., 2012) and truck driver scheduling problems (Goel, Archetti, & Savelsbergh, 2012). The major advantage of these constructive heuristics is that they can yield rosters rapidly. However, the quality of the obtained rosters is often not as good as expected, especially for large-scale problems.

In contrast, the improvement heuristic method begins with an initial roster and then a scheme is implemented for iteratively generating an improved roster (Ying & Liao, 2004). In improvement heuristics, meta-heuristics form an important methodology for the solving of complex PSPs. In the literature, Tabu search (Di Gaspero et al., 2007; Elizondo et al., 2010; Lucic & Teodorovic, 2007), genetic algorithms (Asensio-Cuesta, Diego-Mas, Canos-Daros, & Andres-Romano, 2012; Beddoe & Petrovic, 2006; Moz & Pato, 2007) and simulated annealing (Akbari, Zandieh, & Dorri, 2013; Cordeau, Laporte, Pasin, & Ropke, 2010; Thompson & Goodale, 2006) are the three meta-heuristics most used in the solving of PSPs. In addition to these three general classes, other popular meta-heuristics have been developed for dealing with PSPs, including the scatter search (Burke, Curtois, Qu, & Vanden Berghe, 2010; Laguna, Casado, & Pacheco, 2005; Maenhout & Vanhoucke, 2010), iterated local search (Bellanti, Carello, Della Croce, & Tadei, 2004; Burke, Curtois, van Draat, van Ommeren, & Post, 2011), variable neighborhood search (Burke, Curtois, Post, Qu, & Veltman, 2008; Burke, Li, & Qu, 2010), particle swarm optimization (Akjiratikarl, Yenradee, & Drake, 2007; Gunther & Nissen, 2010; Nissen & Gunther, 2009), memetic algorithm (Ozcan, 2005), electromagnetic meta-heuristic (Maenhout & Vanhoucke, 2007), neural network (Hao, Lai, & Tan, 2004), ant colony optimization (Gutjahr & Rauner, 2007), greedy random adaptive search procedure (Goodman, Dowsland, & Thompson, 2009), hill-climbing heuristic (Cipriano, Di Gaspero, & Dovier, 2006) and the hyper-heuristic algorithm (Chakhlevitch & Cowling, 2005). The major advantage of these meta-heuristics lies in their effectiveness, i.e., their ability to obtain workable rosters within a limited computing time. However, these meta-heuristics cannot demonstrably produce optimal solutions, nor can they demonstrably reduce the search space (Burke, Li, et al., 2010). For further detailed discussion of the available algorithms proposed by the research community with respect to PSPs, the reader is referred to the review articles of Alfares (2004), Burke, De Causmaecker, Vanden Berghe, and Van Landeghem (2004), Kohl and Karisch (2004), Ernst, Jiang, Krishnamoorthy, Owens, and Sier (2004), Ernst, Jiang, Krishnamoorthy, and Sier (2004) and Van den Bergh et al. (2013).

Motivated by the need for practical day-to-day rostering in large service operations, this work has investigated a variant of the PSP, named the *shift minimization personnel task scheduling problem* (SMPTSP). The characteristics of the SMPTSP are: (1) that the only cost incurred is that owing to the number of workers (shifts) that are required to perform the given set of tasks; (2) that each worker possesses the set of qualifications or skills required in the performance of a subset of, but not all, tasks; (3) that each task is performed by exactly one qualified worker, as indicated by a shift with specified start and end times; and (4) that the objective is to minimize the total number of shifts/workers used. This variant of the PSP is important both as a planning tool, in order to minimize the number of staff members that an organization is required to maintain, and as a day-to-day operational management tool, in order to determine the number of staff members needed to perform all tasks on a particular day.

The SMPTSP introduced by Krishnamoorthy, Ernst, and Baatar (2012) was originally derived from a staff rostering problem (Dowling, Krishnamoorthy, Mackenzie, & Sier, 1997) at a large international airport, involving about 500 staff members and a monthly planning horizon. In that study, the authors described a mathematical formulation and proposed a new heuristic based on the *volume algorithm* (VA) and *Wedelin's algorithm* (WA), named VAWA, for solving the SMPTSP. Both the VA and WA allow for some of the harder constraints to be relaxed and, thus, the remaining problem to be easily solved. Therefore, the VAWA is computationally efficient and can be used to solve large-scale problems, such as those found in real-world applications.

In the literature, solution algorithms, with respect to PSPs, are heavily skewed towards exact methods and meta-heuristics. Recently, multi-phase approaches and hybrid techniques, which deal with heavily constrained PSPs, have received increased attention from researchers. Therefore, in this study, an effective and efficient three-phase algorithm has been proposed, which combines the respective advantages of a constructive heuristic, a meta-heuristic and an exact method, for solving the SMPTSP. The remainder of this paper is organized as follows: the SMPTSP is defined in Section 2; in Section 3, the proposed three-phase algorithm is described in detail; in Section 4, the effectiveness and efficiency of the proposed algorithm is evaluated using a test problem set and its performance is compared to an existing algorithm, VAWA, drawn from the literature; and finally, in Section 5, this study concludes with recommendations for future studies.

2. Problem definition

The *shift minimization personnel task scheduling problem* (SMPTSP) can be defined formally as follows. A set of tasks $T = \{t_1, \ldots, t_n\}$ needs to be allocated to a set of heterogeneous workers $W = \{w_1, \ldots, w_m\}$ over a specified planning horizon. The processing time interval at which a task has to be performed is determined by a timetable with fixed start and end times. Each worker possesses a number of specific skills or qualifications which enable them to perform a subset of the tasks, but not all of them. The objective is to find an optimal roster in order to minimize the number of shifts/workers required to perform the given set of tasks. Note that the term 'workers' may also stand for other resources, such as processors or machines. The following assumptions were made for the SMPTSP considered in this study:

- The number of tasks, as well as their start and end times, are fixed and known in advance.
- Each task is considered an indivisible entity, even though it may be composed of a number of individual units.
- Preemption of tasks is not allowed.
- There are no precedence constraints among the tasks.
- Tasks cannot be carried out by workers who do not possess the specific skills required.
- Each task is processed once by a qualified worker.
- Each worker can execute, at most, one assigned task at a time.
- Each worker operates independently of other workers.
- Once a worker starts to perform a task, he/she is continuously available throughout the job completion, and there are no interruptions owing to break time or other such causes.
- Once a task is initiated by a worker, it cannot be transferred to another worker as the entire operation must be completed by the same worker.

The SMPTSP described above can be formulated as the following BIP model, which is similar to a model depicted in Krishnamoorthy et al. (2012):