European Journal of Operational Research 236 (2014) 789–799

Contents lists available at SciVerse ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Vehicle routing with soft time windows and stochastic travel times: A column generation and branch-and-price solution approach

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ARTICLE INFO

Article history:
Received 13 September 2012
Accepted 15 May 2013
Available online 25 May 2013

Keywords:
Routing
Stochastic travel times
Column generation
Branch-and-price

ABSTRACT

We study a vehicle routing problem with soft time windows and stochastic travel times. In this problem, we consider stochastic travel times to obtain routes which are both efficient and reliable. In our problem setting, soft time windows allow early and late servicing at customers by incurring some penalty costs. The objective is to minimize the sum of transportation costs and service costs. Transportation costs result from three elements which are the total distance traveled, the number of vehicles used and the total expected overtime of the drivers. Service costs are incurred for early and late arrivals; these correspond to time-window violations at the customers. We apply a column generation procedure to solve this problem. The master problem can be modeled as a classical set partitioning problem. The pricing subproblem, for each vehicle, corresponds to an elementary shortest path problem with resource constraints. To generate an integer solution, we embed our column generation procedure within a branch-and-price method. Computational results obtained by experimenting with well-known problem instances are reported.

1. Introduction

The Vehicle Routing Problem (VRP), sometimes referred to as capacitated VRP, aims to find a set of feasible routes that start and end at the depot to serve a set of customers. Each customer, given with a known demand, is visited exactly once by one vehicle. Each route can service a total demand that cannot exceed the vehicle capacity. The objective is to minimize the total cost, traditionally derived from the sum of distances traveled or the number of vehicles used or a combination of these. The interested reader is referred to Toth and Vigo [32], and Laporte [22,23] for comprehensive literature surveys about the VRP. This problem is extended by considering different customer service aspects such as starting the service at each customer within a given time interval, called the Vehicle Routing Problem with Time Windows (VRPTW). Time windows are called soft when they can be violated with some penalty costs. They are called hard when violations are not permitted, i.e., vehicles are allowed to wait with no cost if they arrive early and they are prohibited to serve if they arrive late. For reviews on the VRPTW, the reader is referred to Bräysy and Gendreau [4,5], Gendreau and Tarantilis [17], and Kallehauge [21].

In the classical formulation of the VRP, all problem elements are deterministic. However, carrier companies have to deal with various types of uncertainty in real-life applications. Service quality may become quite poor if uncertainties are disregarded at the planning level, since routes may be inefficient or even infeasible in some cases. To overcome the inefficiency incurred at the operational level, stochastic variants of the VRP have been introduced (see Gendreau et al. [16] for a review on stochastic routing problems). Common parameters considered in these variants are stochastic demands, stochastic customers and stochastic travel times. In this research, we study a version of the VRP where we focus on stochastic travel times with a known probability distribution. Using stochastic travel times enables us to construct both reliable and efficient routes. In addition to the cost effectiveness, we also consider customer service aspects where each customer has a soft time window that allows early and late servicing.

The motivation for focusing on the problem described in this paper is that in real-life contexts vehicles operate on a traffic that may be congested, which leads to uncertain travel times. We consider these stochastic travel times at the planning level to accurately evaluate arrivals of vehicles at customer locations by means of stochastic performance measures incorporated in our problem. In practice, these evaluations are carried out with respect to the customers' delivery time intervals. The latter are potentially soft in real-life applications since travel times are uncertain and thus cannot exactly be predicted. Note that customers can often be provided service outside their time windows.

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http://dx.doi.org/10.1016/j.ejor.2013.05.024

For our problem, we consider the formulation introduced in Taş et al. [30], where the authors focus on modeling aspects and on solving the problem effectively with a new solution procedure based on metaheuristics. The study conducted in [30] extends existing models, which are generated for stochastic routing problems, by proposing a one-stage formulation in which the objective function copes with time-window violations and expected overtime, and all constraints are linear (see Ando and Taniguchi [1], Russell and Urban [27], and Li et al. [24] for existing models). In this formulation, the objective is to minimize the sum of transportation costs and service costs. Transportation costs result from three elements which are the total distance traveled, the number of vehicles used and the total expected overtime of the drivers. Service costs are incurred for early and late arrivals; these correspond to time-window violations at the customers. Various solution options can be provided to carrier companies by generating different combinations of two main cost components. In our study, we optimally solve this model.

As mentioned above, the formulation introduced by Taş et al. [30] divides the cost components into transportation costs and service costs. It is important to consider this distinction for real-life applications since carrier companies focus on aspects which are usually different from those that concern their customers. In practice, service providers aim to serve all customers by employing the least (operational) cost vehicle routes. For customers, deliveries need to be satisfied by these routes as reliably as possible (within the predefined time windows as promised). The transportation cost component includes true costs paid by carrier companies. Note that by definition, this component is directly related to efficiency of operations. Service costs are obtained by confronting arrival times with soft time windows at customers and thus directly associated with reliability of operations. In this paper, the uncertainty in travel times is taken into account in both cost calculations.

To obtain the optimal solution of our model, we apply a column generation procedure (see Lübbecke and Desrosiers [25], and Desaulniers et al. [9] for comprehensive surveys on column generation). In our procedure, the master problem can be modeled as a classical set partitioning problem. The pricing subproblem, for each vehicle, corresponds to an Elementary Shortest Path Problem with Resource Constraints (ESPPRC). This column generation procedure is embedded within a branch-and-price scheme to obtain integer solutions. Branch-and-price method has proved to be a very successful exact approach for tackling deterministic and stochastic variants of vehicle routing problems. Some applications can be found in Desrochers et al. [11], Fischetti et al. [15], Chabrier [6], Irnich and Villeneuve [19], and Christiansen and Lysgaard [7]. As far as we know, no one has yet studied exact methods to solve the VRP with soft time windows and stochastic travel times. Our paper extends the related literature by providing such a procedure.

The remainder of this paper is organized as follows. The problem and the formulation used in this paper are introduced in Section 2. The column generation procedure and its pricing problem are presented respectively in Sections 3 and 4. Then, we describe our branch-and-price algorithm in Section 5 and report numerical results on Solomon's problem instances [29] in Section 6. Finally, we provide our conclusions in Section 7.

2. Problem description and formulation

Let $G = (N,A)$ be a connected digraph where $N = \{0, 1, \ldots, n\}$ is the set of nodes and A is the set of arcs. In this graph, node 0 denotes the depot, and nodes 1 to n represent customers. A distance d_{ij}, and a travel time T_{ij} with a known probability distribution are defined for each arc (i,j), where $i \neq j$. With each customer $i \in N\backslash\{0\}$ is associated a positive demand q_i, a positive service time s_i, and a soft time window $[l_i,u_i]$ where l_i and u_i are non-negative parameters. Soft time windows enable to serve customers outside their time windows, but some penalty costs must be incurred for the company for early or late servicing. The scheduling horizon for the problem is represented by $[l_0,u_0]$, which is the time window given for the depot. Furthermore, a homogeneous fleet of vehicles of equal capacity (Q) is located at the depot. These vehicles, which belong to set V, are not allowed to wait at customer locations in case of early arrival; service must take place immediately.

In this paper, we focus on the mathematical formulation introduced in Taş et al. [30]. We first summarize the notation used in this formulation in Table 1.

The model, which is solved by applying exact algorithms in our paper, can then be stated as follows:

$$\min \sum_{v\in V}\left[\rho\frac{1}{C_1}\left(c_d\sum_{j\in N}D_{jv}(\mathbf{x})+c_e\sum_{j\in N}E_{jv}(\mathbf{x})\right)\right.$$
$$\left.+(1-\rho)\frac{1}{C_2}\left(c_t\sum_{i\in N}\sum_{j\in N}d_{ij}x_{ijv}+c_f\sum_{j\in N\backslash\{0\}}x_{0jv}+c_oO_v(\mathbf{x})\right)\right] \quad (1)$$

$$\text{s.t.} \quad \sum_{j\in N}\sum_{v\in V}x_{ijv}=1, \qquad i\in N\setminus\{0\}, \quad (2)$$

$$\sum_{i\in N}x_{ikv}-\sum_{j\in N}x_{kjv}=0, \qquad k\in N\setminus\{0\}, v\in V, \quad (3)$$

$$\sum_{j\in N}x_{0jv}=1, \qquad v\in V, \quad (4)$$

$$\sum_{i\in N}x_{i0v}=1, \qquad v\in V, \quad (5)$$

$$\sum_{i\in N\backslash\{0\}}q_i\sum_{j\in N}x_{ijv}\leqslant Q, \qquad v\in V, \quad (6)$$

$$\sum_{i\in B}\sum_{j\in B}x_{ijv}\leqslant |B|-1, \qquad B\subseteq N\setminus\{0\}, v\in V, \quad (7)$$

$$x_{ijv}\in\{0,1\}, \qquad i\in N,\ j\in N,\ v\in V. \quad (8)$$

The objective function (1) minimizes the total weighted cost which has two main components, service costs and transportation costs. Constraints (2) ensure that each customer is visited exactly once. Constraints (3) are the flow conservation constraints at each customer for each vehicle. Constraints (4) and (5) indicate that every vehicle route must start from and end at the depot. Constraints (6) state that the load of each vehicle cannot exceed its capacity. Constraints (7) are subtour elimination constraints, and (8) are the integrality constraints. Parameter ρ is needed to obtain

Table 1
Notation used in the mathematical model.

x_{ijv}	Equal to 1 if vehicle v covers arc (i,j), 0 otherwise
$\mathbf{x}$	Vector of vehicle assignments and customer sequences in these Vehicle routes, where $\mathbf{x} = \{x_{ijv} \mid i,j \in N, v \in V\}$
$D_{jv}(\mathbf{x})$	*Expected* delay at node j when it is served by vehicle v
$E_{jv}(\mathbf{x})$	*Expected* earliness at node j when it is served by vehicle v
$O_v(\mathbf{x})$	*Expected* overtime of the driver working on route of vehicle v
c_d	Penalty cost paid for one unit of delay
c_e	Penalty cost paid for one unit of earliness
c_t	Cost paid for one unit of distance
c_o	Cost paid for one unit of overtime
c_f	Fixed cost paid for each vehicle used for servicing