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A branch-and-cut-and-price algorithm for the cumulative capacitated vehicle routing problem

Jens Lysgaard *,[1], Sanne Wøhlk [1]

CORAL, Department of Economics and Business, Aarhus University, Fuglesangs Allé 4, DK-8210 Århus, Denmark

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ABSTRACT

In this paper we consider the Cumulative Capacitated Vehicle Routing Problem (CCVRP), which is a variation of the well-known Capacitated Vehicle Routing Problem (CVRP). In this problem, the traditional objective of minimizing total distance or time traveled by the vehicles is replaced by minimizing the sum of arrival times at the customers. We propose a branch-and-cut-and-price algorithm for obtaining optimal solutions to the problem. To the best of our knowledge, this is the first published exact algorithm for the CCVRP. We present computational results based on a set of standard CVRP benchmarks and investigate the effect of modifying the number of vehicles available.

1. Introduction

The Capacitated Vehicle Routing Problem (CVRP) is one of the most well-studied problems within the area of transportation optimization. Recently it has, however, become clear to the research community that the CVRP does not fully capture the essence of real life transportation problems. This has for instance led to the introduction of so-called Rich Vehicle Routing Problems (Hartl, Hasle, & Janssens, 2006), a family of problems which capture the complications of real life problems far better than the classical CVRP.

One aspect of rich vehicle routing problems is the consideration of objective functions that differ from the traditional one of minimizing the total distance or time traveled by the vehicles. These include minimizing the number of vehicles used and minimizing the length of the longest tour. The latter is known as a Min–Max objective and is studied by Golden, Laporte, and Taillard (1997) and Applegate, Cook, Dash, and Rohe (2002), among others. Other studies consider simultaneous optimization of multiple objectives (Bowerman, Hall, & Calamai, 1995; Corberán, Fernández, Laguna, & Martí, 2002).

In this paper we consider the variation of the CVRP where the objective is to minimize the sum of arrival times at the customers, for a fixed starting time of each route. This problem is known as the Cumulative Capacitated Vehicle Routing Problem (CCVRP). We note that minimizing the sum of arrival times is equivalent to minimizing the average arrival time.

The CCVRP occurs in several applications. It is relevant in distribution systems where it is desirable to provide early service measured across the whole set of customers. In school bus routing, for example, minimizing average arrival time is one fairness measure which may have priority over minimizing total distance traveled. A more detailed discussion of performance criteria in the context of school bus routing is provided in Bowerman et al. (1995). Furthermore, when natural disasters strike, it is essential that aid arrives quickly in order to save lives and provide emergency supplies, so the traditional goal of cost minimization must step aside for fast response and fairness. Several performance measures can be used in relation to providing aid to multiple locations quickly, and minimizing the latest arrival time or minimizing the average arrival time are among the commonly used. The effect on the quality of one objective function when optimizing another is investigated by Campbell, Vandenbussche, and Hermann (2008) in the context of relief effort.

The outline of our paper is as follows. We first consider related literature in Section 2. Our mathematical model formulation is presented in Section 3, our algorithm is described in Section 4, and computational results are given in Section 5. Finally, we present the conclusion and perspectives in Section 6.

2. Related literature

The CCVRP has recently been studied from a heuristic point of view in several papers. These studies include Iterated Local Search (Chen, Dong, & Niu, 2012), Adaptive Large Neighborhood Search (Ribeiro & Laporte, 2012), Memetic algorithms (Ngueveu, Prins, & Wolfler Calvo, 2010), and a two-phase heuristic (Ke & Feng, 2013). In Ke and Feng (2013), the performance of three algorithms

* Corresponding author. Tel.: +45 8716 4898.
E-mail addresses: lys@asb.dk (J. Lysgaard), sanw@asb.dk (S. Wøhlk).

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(Ke & Feng, 2013; Ngueveu et al., 2010; Ribeiro & Laporte, 2012) were compared. Based on that comparison, the two-phase algorithm (Ke & Feng, 2013) and the Adaptive Large Neighborhood Search (Ribeiro & Laporte, 2012) each provide the best known solution for about half of the test instances.

Kara, Kara, and Yetiş (2008) consider a variation of the CCVRP, where the objective function to be minimized is not the sum of arrival times, but rather the sum of arrival times multiplied by the demand of the node. They refer to this problem, which is an extension of the CCVRP, as CumVRP and study the relationship to various other problems. Flow based formulations of the problem for both the delivery and the collection case are presented and based on these, the authors are able to solve instances with up to 34 locations.

The uncapacitated version of the CCVRP is known as the k-traveling repairman problem, where k is the number of vehicles available. For this problem, Fakcharoenphol, Harrelson, and Rao (2007) present an 8.497-approximation algorithm which is partly based on a result due to Chaudhuri, Godfrey, Rao, and Talwar (2003). Jothi and Raghavachari (2007) study an extension of the k-traveling repairman problem in which there is a repair time in addition to the travel time. They present a $(\frac{3}{2}\beta+\frac{1}{2})$-approximation algorithm for this problem, where β is the approximation factor obtainable for the k-traveling repairman problem.

The related single vehicle problem, referred to as the Minimum Latency Problem (MLP), has attracted many researchers. It is well studied both from an approximation and an exact point of view. The current best approximation algorithm for the MLP is due to Chaudhuri et al. (2003) and achieves an approximation factor of 3.59. Several exact approaches have been proposed for the MLP, most of which are based on dynamic programming, branch-and-bound, or a combination of the two (Wu, Huang, & Zhan, 2004). We refer the reader to Silva, Subramanian, Vidal, and Ochi (2012) for an overview of research on the MLP.

Dewilde, Cattrysse, Coene, Spieksma, and Vansteenwegen (2013) study a single vehicle problem, where a profit is obtained the first time a location is visited and the visit of each location is optional. The objective is to maximize the sum of profits minus the sum of arrival times. This problem arises as the subproblem if the CCVRP is solved using column generation approaches, except from the exclusion of capacity constraints. The authors present a tabu seach algorithm for this problem.

Recently, some interesting problem variations, where the MLP is combined with another problem, have been studied. We point out a couple of such papers. Levin and Penn (2008) present a 16.31-approximation algorithm for a combination of MLP and machine scheduling where n jobs are to be processed on a single machine located at a plant and subsequently are to be delivered to n individual customer locations by a single vehicle. Processing times are given for the jobs and travel times are given among the customers and between the plant and the customers. The goal is to determine a production sequence for the jobs at the machine and determine the routing of the vehicle such that the sum of the delivery times of the jobs at the customers are minimized. We emphasize that it is fully allowed for the vehicle to pick up jobs from the plant several times. Li, Vairaktarakis, and Lee (2005) consider a variation of the same problem where several jobs can be associated with the same customer and the vehicle may or may not be capacity constrained.

Chakrabarty and Swamy (2011) study a combination of MLP and facility location. Given a central depot, a set of customers, and a set of possible facilities, the problem is to determine which facilities to activate. The objective function to be minimized is combined of three terms: a fixed cost for each activated facility, a cost of assigning each customer to a facility (this is not a routing cost), and a minimum latency cost of a tour connecting the depot to the facilities.

3. Model formulation

The CCVRP can be defined as follows. Let $G=(V, E)$ be a complete undirected graph, with $V=\{0, \ldots, n\}$. Vertex 0 represents a depot, whereas each of the vertices in $V_c=\{1, \ldots, n\}$ represents a customer. The symmetric travel time between vertices i and j is denoted by t_{ij}. A number K of identical vehicles, each of capacity $Q>0$, is available. Each customer i has an integer demand q_i, with $0<q_i\leqslant Q$. Each customer must be served by a single vehicle and no vehicle can serve a set of customers whose demand exceeds its capacity. Each vehicle used must leave the depot at time 0, visit one or more customers, and return to the depot. The objective is to minimize the sum of all n arrival times at the customers.

In the following subsections we first describe two individual formulations (a Set Partitioning and a Vehicle Flow formulation, respectively) which are then combined into the formulation that we solve by our algorithm.

3.1. A Set Partitioning formulation

We define a feasible *elementary* route as a path $(0, z_1, \ldots, z_k, 0)$, where $z_1, \ldots, z_k$ are k different customers whose total demand does not exceed the vehicle capacity Q. As such, any feasible elementary route starts and ends at the depot, and we use the convention that the starting time at the depot is zero for any route.

For any feasible elementary route r, we define its cost c_r as the sum of arrival times for all customers on the route. Further, let $\Re$ denote the set of all feasible elementary routes. Moreover, we let α_{ir} be a parameter of value 1 if route r visits customer i and 0 otherwise, and we let λ_r be a variable of value 1 if route r is chosen and 0 otherwise. This leads to the following Set Partitioning formulation:

(SPP)

$$\min: \quad \sum_{r\in\Re} c_r\lambda_r \tag{1}$$

$$\text{s.t.}: \quad \sum_{r\in\Re} \alpha_{ir}\lambda_r = 1 \quad \forall i\in V_c \tag{2}$$

$$\sum_{r\in\Re} \lambda_r = K \tag{3}$$

$$\lambda_r \in \{0,1\} \quad \forall r\in\Re \tag{4}$$

The objective (1) minimizes the total cost of all routes. Constraints (2) ensure that each customer is contained in exactly one route, the constraint (3) specifies the required number of routes K, and (4) are the binary constraints on the decision variables.

This model can be solved by branch-and-bound (BB), where the Linear Programming (LP) relaxation in each subproblem is solved by column generation (CG). This would result in a branch-and-price (BP) algorithm where the CG subproblem is the problem of determining a feasible elementary route of minimum reduced cost, for a given set of dual prices associated with (2) and (3).

In this paper, however, we will develop a different formulation involving the same set of route variables, and apply branch-and-cut-and-price (BCP) on this formulation.

3.2. A vehicle flow formulation

While any feasible solution to the CCVRP can be described in terms of λ-variables as in (2)–(4), we also have the alternative of representing a solution as in the two-index solution space used in vehicle flow formulations of the CVRP (Laporte, 2009; Lysgaard, Letchford, & Eglese, 2004), which we reproduce here with the objective function intentionally omitted. Let x_{ij} denote the number of times a vehicle travels directly between vertices i and j. Moreover, for any $S\subset V$, let $\delta(S)$ denote the set of edges with exactly one end-vertex in S, where we for simplicity write $\delta(i)$ instead of