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# The directed profitable location Rural Postman Problem

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ABSTRACT

In this paper we introduce an extension of the well known Rural Postman Problem, which combines arc routing with profits and facility location. Profitable arcs must be selected, facilities located at both end-points of the selected arcs, and a tour identified so as to maximize the difference between the profit collected along the arcs and the cost of traversing the arcs and installing the facilities. We analyze properties of the problem, present a mathematical programming formulation and a branch-and-cut algorithm. In an extensive computational experience the algorithm could solve instances with up to 140 vertices and 190 arcs and up to 50 vertices and 203 arcs.



## 1. Introduction

In this paper, we present a model to address a network design problem that calls for selecting, in a way that is economically sound, terminals and asymmetric connections. Given a directed graph with a set of profitable arcs, with fixed costs associated with end-points, the problem consists in choosing a subset of profitable arcs in such a way that the difference between the profit collected and the total cost is maximized. The total cost includes the cost to open facilities at the end-points of the selected arcs and the traveling cost. We assume that enough capacity is available to satisfy the demand that originates from the selected connections. This assumption can however be removed by introducing an adequate constraint in the integer programming model. We call this problem the *Directed Profitable Location Rural Postman Problem* (DPLRPP).

To the best of our knowledge, the DPLRPP was never studied in the literature. However, strongly related problems were addressed, like the *Eulerian Location Problems* (ELP), treated in Ghiani and Laporte (1999), and the *Location-Arc Routing Problems* (L-ARP), studied in Ghiani and Laporte (2001) and Liu, Jiang, Chen, Liu, and Liu (2008): these are arc routing problems with costs located at nodes and connections, but *without profits*. In Ghiani and Laporte (2001) and Liu et al. (2008), the authors present applications of the L-ARP to those routing and delivering problems that require specific facilities like vehicle depots, relay boxes, dump sites, or transfer/replenishment points. In the DPLRPP costs are located at both nodes and connections, like in ELP and L-ARP. However, contrary to ELP and L-ARP, the problem is *with profits*.

The application domain is mid/long range passenger/freight transportation, e.g., long-distance coach services, airlines, and interstate trucking. In these cases, companies identify a set of potential connections on the basis of demand estimate. The problem is to choose the connections on which to open a service. Unlike local or regional services, and unlike (Ghiani & Laporte, 1999, 2001; Liu et al., 2008), an important characteristic of mid/long range applications is that the company has to bear arc costs, related to transportation service provision, and also node costs, related to passenger/freight service (ticketing, check-in, waiting rooms, warehousing etc.) at *both* terminals. Terminals and connections form a network, the features of which depend on the specific domain. In the case of interstate freight transportation (e.g., the European TIR, Transports Internationaux Routiers) connections are typically asymmetric, because asymmetry is introduced by import–export imbalances.

The paper is organized as follows. We first describe the problem in Section 2, provide an example of application and review the literature. In Section 3 we state the complexity of the problem and survey a few basic properties while in Section 4 we present an integer linear programming formulation with a number of constraints that grows exponentially with $|V|$. Based on this formulation, a branch-and-cut algorithm is then presented together with computational results in Section 5.

## 2. The problem

The DPLRPP is an extension of the well known Rural Postman Problem that combines facility location and arc routing with profits, and is defined as follows.

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Let $G = (V, A)$ be a directed connected graph with $V = \{0, 1, \ldots, n\}$ and $A \subseteq V \times V$ vertex and arc set, respectively. Vertex 0 is the depot. Each vertex $v \in V$ is associated with the cost $f_v$ of installing a facility in $v$. Each arc $(u, v) \in A$ is associated with the (non-negative) cost $c_{uv}$ of traversing $(u, v)$. Moreover, a subset of *profitable arcs* $R \subseteq A$ is defined. Each of these arcs is associated with a non-negative profit $p_{uv}$ and an integer demand $d_{uv}$: the profit is gained only if $(u, v)$ is traversed at least $d_{uv}$ times.

A tour on $G$ is a sequence of arcs $T = ((0, u), (u, v), \ldots, (w, 0))$ starting and ending at 0. The sequence is not necessarily simple, that is, $T$ may contain repeated copies of the same arc of $G$. If the tour traverses $(u, v) \in R$ at least $d_{uv}$ times, then profit $p_{uv}$ is gained. However, unlike costs, which are paid whenever an arc is traversed, a profit can be collected only once. Also, profits are asymmetric, that is, if $u$ and $v$ are connected in both directions, the profit gained when traversing $(u, v)$ may differ from that gained when traversing $(v, u)$. In the following, we use the term 'served' to indicate a profitable arc $(u, v)$ that is traversed at least $d_{uv}$ times. In order for $T$ to be feasible, facilities must be located at each vertex incident to a profitable arc served by $T$. Let $U(T) \subseteq V$ be the set of these vertices.

The DPLRPP is the problem of finding a tour $T$ that maximizes the total net profit $\bar{p}(T)$ given by the difference between the profit $p(T)$ returned from the arcs of $R$ served by $T$ and the cost of $T$ (including both traveling and fixed costs):

$$\bar{p}(T) = p(T) - \sum_{(u,v) \in T} c_{uv} - \sum_{v \in U(T)} f_v \qquad (1)$$

### 2.1. An example

Consider a company interested in offering transportation facilities and services to a set of potential customers, e.g. industries, scattered in a geographical area over a given time period. Suppose that the company is based in Bremen.

Let $V$ be the set of all relevant nodes of the area. Vertex 0 corresponds to the city of Bremen. The potential customers are located at nodes $u \in V$ and express a need to deliver freight to their own customers located at nodes $v \in V$. Freight has to be delivered $d_{uv}$ times from $u$ to $v$. We say that the potential request of service from $u$ to $v$ has demand $d_{uv}$. A profit $p_{uv}$ is gained if the demand is served, that is if the freight is transported from $u$ to $v$ the requested number of times. A profitable arc is associated with each of these potential requests of service. We indicate by $R$ the set of all profitable arcs. The set of arcs $A$ is formed by all oriented links $(u, v)$ which define connections between pairs of nodes, that is links a truck can travel. The set of arcs $A$ contains the set of profitable arcs $R$ and other non-profitable arcs. In practice, profitable and non-profitable arcs are selected from a road map filtering inadequate (e.g., too long or too expensive) connections.

The leftmost table in Fig. 1 gives a set of potential requests of service, with the associated profit $p_{uv}$ and demand $d_{uv}$. Using real travel times and filtering off connections requiring more than $T_{max}$ = 360 minutes, we obtain the graph of Fig. 2. For ease of representation, in Fig. 2 we drew profitable links as directed arcs. Non-profitable connections are represented by edges meaning that the connection between the corresponding pair of cities can be traversed in both directions. Note that profitable arcs can also be traversed in the opposite direction.

In order to make a business plan, the company wants to find the most profitable tour in $G$. The elements of this tour – terminals and connections – are chosen on the basis of costs and profits.

The most important cost sources, measured on a long-term base, are of two types:

- costs $f_v$ allocated to terminals, to be covered in order to provide terminal services, such as warehousing: these costs may include infrastructure availability (e.g., premises rental) and fixed operational costs, and do not depend on the number of connections served;
- mileage-toll costs $c_{uv}$ allocated to links, depending on the link length and on highway tariffs. Mileage costs not only include fuel consumption, but also the relevant quote of driver/hours, fleet maintenance, etc.

Costs $c_{uv}$ are estimated on the basis of the length of the link $(u, v)$ and reported in Fig. 3. The cost $f_v$ of setting up a terminal at $v$ is indicated for each city in the rightmost table of Fig. 1.

Fig. 4 shows the optimal tour that gives a net profit estimate of 1,797,472 euros, over a total cost of 5,083,137 euros (2,351,000 for terminal set-up and 2,732,137 for routing). The solution selects 18 profitable arcs out of 22 (82%). This solution has been found through the branch-and-cut algorithm described in Section 5.

### 2.2. Literature review

The class of problems studied in the literature which is most closely related to the DPLRPP is the class of LOCATION-ARC ROUTING PROBLEMS (L-ARP) (Ghiani & Laporte, 2001; Liu et al., 2008), where a set of required edges have to be served by vehicles starting and ending

| Request of service | Profit | Demand |
|---|---|---|
| Antwerpen - Le Mans | 85,320 | 1 |
| Barcelona - Lyon | 40,968 | 1 |
| Barcelona - Zaragoza | 158,256 | 2 |
| Bordeaux - Barcelona | 176,400 | 2 |
| Bordeaux - Le Mans | 92,160 | 1 |
| Bratislava - Trieste | 111,024 | 2 |
| Bratislava - Zagreb | 152,640 | 3 |
| Brno - Zagreb | 84,456 | 1 |
| Düsseldorf - Zürich | 85,248 | 1 |
| Eindhoven - Düsseldorf | 71,496 | 1 |
| Erfurt - Bremen | 226,080 | 3 |
| København - Bremen | 23,544 | 2 |
| Le Mans - Lyon | 81,000 | 1 |
| Lyon - Marseille | 49,392 | 1 |
| Milano - Roma | 226,224 | 3 |
| Milano - Zürich | 90,864 | 2 |
| Napoli - Roma | 227,952 | 3 |
| Praha - München | 160,344 | 2 |
| Roma - Napoli | 84,960 | 2 |
| Trieste - München | 156,312 | 2 |
| Zagreb - Bratislava | 82,800 | 2 |
| Zürich - Milano | 157,032 | 3 |

| City | Terminal cost (thousands of euros) |
|---|---|
| Antwerpen | 120 |
| Barcelona | 122 |
| Bordeaux | 88 |
| Bratislava | 62 |
| Bremen | 160 |
| Brno | 54 |
| Düsseldorf | 145 |
| Eindhoven | 143 |
| Erfurt | 95 |
| København | 130 |
| Le Mans | 160 |
| Lyon | 180 |
| Marseille | 123 |
| Milano | 175 |
| München | 195 |
| Napoli | 98 |
| Praha | 67 |
| Roma | 110 |
| Trieste | 86 |
| Zagreb | 48 |
| Zaragoza | 67 |
| Zürich | 192 |

**Fig. 1.** Profits $p_{uv}$ and frequency $d_{uv}$ per request of service, and terminal costs.