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A branch-and-cut-and-price approach for the pickup and delivery problem with shuttle routes

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ABSTRACT

The Pickup and Delivery Problem with Shuttle routes (PDPS) is a special case of the Pickup and Delivery Problem with Time Windows (PDPTW) where the trips between the pickup points and the delivery points can be decomposed into two legs. The first leg visits only pickup points and ends at some delivery point. The second leg is a direct trip – called a shuttle – between two delivery points. This optimization problem has practical applications in the transportation of people between a large set of pickup points and a restricted set of delivery points.

This paper proposes three mathematical models for the PDPS and a branch-and-cut-and-price algorithm to solve it. The pricing sub-problem, an Elementary Shortest Path Problem with Resource Constraints (ESPPRC), is solved with a labeling algorithm enhanced with efficient dominance rules. Three families of valid inequalities are used to strengthen the quality of linear relaxations. The method is evaluated on generated and real-world instances containing up to 193 transportation requests. Instances with up to 87 customers are solved to optimality within a computation time of one hour.

The Pickup and Delivery Problem (PDP) consists in defining a set of minimum cost routes to serve a set of independent transportation requests. Each transportation request is defined by an origin called the *pickup point*, a destination called the *delivery point* and a positive value called demand or weight. The PDP is one of the many extensions of the Vehicle Routing Problem (VRP). If time windows for requests are considered, the problem is called the Pickup and Delivery Problem with Time Windows (PDPTW). The demand may be static (each request is known in advance) or dynamic (new requests may arrive at any time). Requests can correspond to the transportation of goods or people. Depending on the context of the application, various objective functions may be considered: number of vehicles used, total ride time, total distance, cost or quality of service–related criteria. There is an abundant literature on pickup and delivery problems and many references can be found in recent surveys by Parragh, Doerner, and Hartl (2006), Berbeglia, Cordeau, Gribkovskaia, and Laporte (2007) and Cordeau, Laporte, and Ropke (2008).

The Pickup and Delivery Problem with Transfers (PDPT) is a recent extension of the PDP relaxing the constraint that the pickup and delivery points of a given request are serviced by the same vehicle. Instead, some requests may be transferred from one vehicle to another at some predetermined intermediate locations called *transfer points*. Then, distinct vehicles pick up the transferred requests and drives them to their respective delivery point. The PDPT implies a new precedence constraint stating that, if a request uses a transfer point, it must be delivered to the transfer point by a first vehicle before being picked up by a second vehicle. The objective function of the PDPT can be the same as that of the PDP.

Since the PDPT introduces hard precedence and synchronization constraints between vehicles, it is much more difficult to solve in practice than the PDP. The exact methods described in the literature can only solve small instances of the PDPT: Cortés, Matamala, and Contardo (2010) report optimal results for instances with 6 requests, Kerivin, Lacroix, Mahjoub, and Quilliot (2008) report optimal results for instances with 15 requests.

In this paper we introduce the Pickup and Delivery Problem with Shuttle routes (PDPS) which is a special case of the PDPT. The PDPS is a problem that arises in passenger transportation when passengers are picked up at a multitude of locations and dropped off at a few, common delivery points. The set of vehicle routes is split into two categories: *pickup routes* and *shuttle routes*. Pickup routes visit a set of pickup points independently of their associated delivery points and end at one delivery point. Shuttle routes are direct trips between two delivery points. Requests can then be transported in one leg (pickup route) or two legs (pickup route + shuttle route) from their pickup point to their delivery point.

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The main contribution of the paper is to introduce the PDPS, to propose an exact algorithm for this problem based on the branch-and-cut-and-price paradigm and to optimally solve realistic-size instances (up to 87 requests) of the PDPT for the first time, albeit a special case.

The remainder of this paper is organized as follows. Section 1 shows a simple example and discusses the potential benefits of transfers. Section 2 summarizes the related works and positions the present paper in the literature. Section 3 formulates the PDPS, introduces a graph modeling of transfer points, and includes one arc-based and two path-based integer linear programming formulations of the problem. Section 4 describes the branch-and-cut-and-price method used to solve the PDPS. Section 5 presents computational experiments on generated instances with 15–75 requests and on real-life instances with 81–193 requests. Section 6 concludes the paper.

1. Benefits of transfers and motivation for the work

One reason to consider the transfer of requests is the potential savings that can be achieved by this practice. This is illustrated by the example given in Fig. 1. We consider 9 requests: requests 1, 2 and 3 share the pickup point p_{123}, requests 4, 5 and 6 share the pickup point p_{456}, requests 7, 8 and 9 share the pickup point p_{789}. Requests 1, 2, 4, 5, 7 and 8 must be delivered to d_{124578} while requests 3, 6, and 9 must be delivered to d_{369}. The nodes o and o' represent the starting depot and the ending depot respectively. The distances between the points are reported in Fig. 1.

We consider that vehicles have a capacity of 3 and assume that routes are doubly-open, meaning that they start at first pickup point and end at the last delivery point. The optimal solution without any transfers uses three vehicles: vehicle 1 picks up requests 1, 2 and 3 at point p_{123} then unloads requests 1 and 2 at d_{124578} and request 3 at d_{369}. Vehicle 2 starts at p_{456}, visits d_{124578} and ends at d_{369}. Vehicle 3 starts at p_{789}, visits d_{124578} and ends at d_{369}. Each route has a cost of 4 so the total routing cost is 12.

If the transfer of passengers is allowed at point d_1, vehicles 2 and 3 can be totally emptied and end their trip at d_1. Then requests 6 and 9 can be loaded into vehicle 1 with request 3. The vehicle follows its itinerary to point d_2. Vehicle 1 has a route cost of 4 while vehicles 2 and 3 have a route cost of 2 so the total routing cost is 8. In this case, the use of a transfer point enables 33% to be saved on the cost of the optimal solution.

However, transfers also have negative impacts which can put the brakes on the practical implementation of such a system. The main drawbacks are the cost of building or running the facility corresponding to the transfer point, the need for tight synchronization between vehicles and the supplementary handling operations at the transfer point. Moreover, in the context of demand-responsive transportation, direct trips are often preferred by passengers. Transfers are seen as a reduction of the quality of the service. Thus,

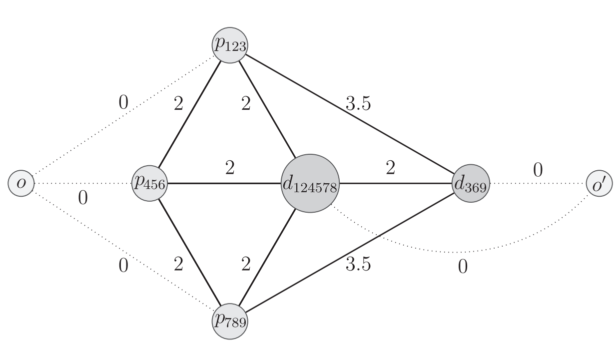

Fig. 1. Example where using transfer point leads to significant savings.

the decision makers have to balance the expected benefits and drawbacks of transfers.

2. Related works and position of the problem

Although the PDP has been intensively studied in the last 15 years, the PDPT has been the subject of very few works. Cortés et al. (2010) present a mathematical formulation of the PDPT that is solved using a branch-and-cut algorithm. Instances with 6 requests and 2 vehicles are solved to optimality. Kerivin et al. (2008) consider a PDP where every request can be split as well as transferred from one vehicle to another at every node of the problem. This problem has no time window and is solved using branch-and-cut. Some instances with up to 15 requests are solved to optimality. Nakao and Nagamochi (2010) calculate a lower bound for the PDPT with a single transfer point. Transfers also appear in the school bus model of Fugenschuh (2009). In this work, the routes are already designated and the objective is to minimize the number of buses needed to cover these routes, allowing some transfer between buses.

As far as heuristic approaches are concerned, Mitrović-Minić and Laporte (2006) propose a local search method for the uncapacitated PDPT with a Manhattan distance. They solve generated instances with 100 requests. An Adaptive Large Neighborhood Search (ALNS) for the PDPT was proposed by Masson, Lehuédé, and Péton (2013a) and applied to the instances of Mitrović-Minić and Laporte (2006) and real-world instances with between 2 and 33 transfer points and between 55 and 193 requests. This method was adapted to the Dial–A–Ride Problem with Transfers in Masson, Lehuédé, and Péton (in press). In this study, the authors underline the complexity of enforcing the maximum ride time constraint.

Petersen and Ropke (2011) present another ALNS to handle freight transportation problems considering one transfer point and up to 982 requests. Qu and Bard (2012) propose a GRASP with an ALNS for solving generated instances with 25 requests and one transfer point. Their method can solve 88% of the instances with a distance no greater than 1% of the optimum.

The use of transfer points has also been considered, with special restrictions, for solving pickup and delivery problems. Russell, Morrel, and Haddox (1986) present a school bus routing application where some children are brought to a transfer point, where they are transferred into shuttle buses that take them to their school. The authors present a heuristic to solve this problem. In this approach, the design of the shuttle routes is carried out as a post-processing step. Shang and Cuff (1996) consider that any point can be used as a transfer point. Transfers are only considered to insert a request that cannot be inserted in the current solution without resorting to an extra vehicle. The heuristic relies on the construction of mini-routes which are assigned to vehicles. Thangiah, Fergany, and Awam (2007) use the same principles in a real-time version of the problem. Lin (2008) presents a PDP with time windows where all requests share the same delivery point but have distinct delivery time windows. In this problem, it is considered that a transfer can occur on the last pickup before a delivery. The author uses a heuristic based on a set-partitioning formulation of the problem to solve instances with up to 100 requests. Oertel (2000) presents a tabu search for a version of the problem with two transfer points. Gørtz, Nagarajan, and Ravi (2009) consider a version of the PDPT where the objective is to minimize the makespan. Approximation algorithms are proposed to solve the uncapacitated and capacitated cases. A heuristic column generation method is proposed by Mues and Pickl (2005). They consider a problem with a single transfer point through which requests are systematically routed. The set of routes is composed of pickup routes ending at the transfer point and delivery routes starting at this transfer point. Instances with up to 70 requests are considered for experiments.