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Ranking paths in stochastic time-dependent networks

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ABSTRACT

In this paper we address optimal routing problems in networks where travel times are both stochastic and time-dependent. In these networks, the best route choice is not necessarily a path, but rather a *time-adaptive strategy* that assigns successors to nodes as a function of time. Nevertheless, in some particular cases an origin–destination path must be chosen a priori, since time-adaptive choices are not allowed. Unfortunately, finding the *a priori* shortest path is an NP-hard problem.

In this paper, we propose a solution method for the a priori shortest path problem, and we show that it can be easily extended to the ranking of the first K shortest paths. Our method exploits the solution of the time-adaptive routing problem as a relaxation of the a priori problem. Computational results are presented showing that, under realistic distributions of travel times and costs, our solution methods are effective and robust.

1. Introduction

Classical optimization models for routing commodities, vehicles, passengers, etc. in a transportation network assume that link travel times are deterministically known and do not evolve over time. In real cases this assumption is often unrealistic, indeed, several different ways of representing uncertainty and/or variability have been proposed in the literature. In this paper we consider *stochastic time-dependent networks* (*STD networks*) where link travel times are represented by random variables with probability distributions varying as a function of departure times. We distinguish between *discrete* and *continuous* STD networks, according to the representation of time and the nature of the random variables. Routing models based on STD networks have been often adopted in application areas such as hazardous material transportation (Chang, Nozick, & Turnquist, 2005), advanced vehicle navigation and traveler information systems (Gao & Huang, 2012; Ardakani & Sun, 2012) and transit passenger path choice (Hickman & Bernstein, 1997).

Optimal routing in STD networks was first addressed by Hall (1986), who considered the minimization of *expected travel time* for a given origin/destination pair and starting time. Hall pointed out two different ways of formulating the problem. If a route must be specified before travel begins, and no deviations are permitted, a simple (i.e., loopless) path must be selected; this is referred to as *a priori route choice*. However, the shortest route is not necessarily a path but rather a *time-adaptive strategy* that assigns optimal successors to a node as a function of leaving time; this is referred to as *time-adaptive route choice*. The adaptive problem turns out to be computationally easy; in a discrete STD-network an optimal strategy can be found in linear time in the size of the network description (Pretolani, 2000; Miller-Hooks, 2001). By contrast, a priori route choice has been shown to be strongly NP-hard also for discrete and deterministic time-dependent networks (Orda & Rom, 1991; Pretolani, 2000).

A solution approach for a priori routing, valid for both discrete and continuous STD networks, was proposed by Hall (1986); his method enumerates loopless paths in a suitably defined deterministic and static version of the STD network. As observed in Miller-Hooks and Mahmassani (2000), Hall's method has relevant potential drawbacks, since it may process all existing paths before termination. For discrete STD networks, Miller-Hooks and Mahmassani (2000) proposed a *labeling* algorithm that finds a priori optimal paths, to a given destination, from all the other nodes and for all possible leaving times. In fact, they allow the paths to be looping, which is a major depart from Hall's original model. This method has similar drawbacks as Hall's method (see Miller-Hooks & Mahmassani, 2000, Proposition 4) but turns out to be quite effective for sparse random networks. For continuous STD networks, Fu and Rilett (1998) proposed a heuristic approach based on a technique for approximating the expected length of a path. Based on similar techniques, an efficient algorithm was devised by Fu (2001), and later extended to the multicriterion case by Chang et al. (2005).

Despite being less flexible and computationally harder, a priori route choice may represent the only alternative in several situations. This is the case, for example, if the traveller does not have access, or is not willing (think of a daily commuter) to react to

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information made available during the travel. Another relevant example is the transportation of highly sensitive substances, where it is necessary to commit in advance to a specific path, that must be preapproved and monitored. Chang et al. (2005) address a priori routing in this context. Moreover, on the strategic level the difference between the minimum cost under a priori and time-adaptive routing may provide an indication of the value of using on-line information. This indication may support the decision of investing in on-board navigation systems and road infrastructures, see e.g. Gao and Huang (2012), Ardakani and Sun (2012), Boyles and Waller (2011) for some related issues. A comparison between a priori and adaptive routing goes beyond the scope of this work; the interested reader may find some results in Miller-Hooks and Mahmassani (2000) and Nielsen (2004).

In many cases where a priori routing is mandatory, finding a single shortest path may not be satisfactory, and it becomes relevant to determine a set of alternative optimal or nearly-optimal paths. This allows e.g. to select the shortest path satisfying some additional constraints not captured by the network model. In other cases we may be interested in selecting a set of *spatially dissimilar* paths, rather than a single one; this typically happens in hazardous materials transportation, in order to equally distribute the risk among the population. A common approach (see e.g. Akgün, Erkut, & Batta, 2000) consists in selecting dissimilar paths out of a (large) set of previously generated "attractive" paths. In the situations above, one needs effective methods for *ranking* a priori paths, that is, the STD counterpart of the classical *K shortest (loopless) paths* in directed graphs (Yen, 1971). However, to the best of our knowledge, ranking of a priori paths in STD networks has never been addressed in the literature, with the exception of Nielsen, Pretolani, and Andersen (2009) where a ranking procedure was embedded in a *two phase method* for bicriterion a priori routing.

In this paper we consider a priori route choice in discrete STD networks, for a single origin, a single destination, and a given departure time. Our goals are to devise solution methods for the shortest and K shortest path problems, and to evaluate their effectiveness and robustness against a set of challenging instances. In the following we describe in detail the aim and contribution of our work. First, it is worthwhile to add a couple of remarks on the model considered here. Since we address a priori route choice, we ignore all kind of on-line information, including arrival times at intermediate nodes, which implies that waiting is forbidden. Furthermore, according to the original model proposed by Hall, we assume that travel times are *independent* random variables. Models of STD networks with *correlated* travel times have been proposed, see e.g. Huang and Gao (2012) for a priori route choice; these models are much more computationally demanding than the classical one.

We devise a *best-first* branch and bound method, where subproblems correspond to subnetworks of the STD network. Due to the best-first policy this method generates paths in non-decreasing order of cost, and thus deals quite naturally with the ranking of a priori paths. A relevant methodological contribution of our approach is that we solve time-adaptive problems as a *relaxation* of a priori problems. The fact that the former problem is a relaxation of the latter has already been pointed out (Miller-Hooks & Mahmassani, 2000), but has never been exploited for algorithmic purposes. Within our method, the use of the adaptive relaxation allows to skip (or at least delay) the processing of unpromising subproblems, thus avoiding the pitfalls of the previously proposed methods. We also devise a version of our method solving time-adaptive subproblems via *reoptimization* (see Nielsen, Pretolani, & Andersen, 2006). This version turns out to be consistently faster.

To the best of our knowledge, our work is the first one addressing the ranking of a priori paths in STD networks, both from a methodological and a computational point of view. It is instructive to point out that previous approaches for the shortest a priori path do not seem suitable for ranking purposes. This may shed light on the motivations and the relevance of our methodological contribution. The method proposed by Hall, despite some similarities to our one, does not necessarily generate paths in nondecreasing order of travel time. Indeed, it generates paths in nondecreasing order of *length*, which is a (quite loose) lower bound on the expected travel time. As a consequence, the path generation stops only when the length of the last generated path meets the Kth best solution found so far, provided this happens before enumerating all the existing paths. Similar and possibly worse drawbacks affect labeling methods, in particular *label correcting* ones. In fact, we do not know how a labeling method may be extended to ranking in STD networks, at least without the addition of suitable bounds or domination rules. Note that the above drawbacks are avoided in our method, due to the best-first policy and the use of the time-adaptive relaxation.

When considering finding the shortest a priori path (i.e., the case $K = 1$) we do not make strong claims on the merits of our approach, since our setting differs substantially from those of previous proposals. Indeed, labeling methods for the discrete case are conceived for a much more general version of the problem; on the other hand, solution methods based on the continuous model are inherently approximate, even if they may offer a better trade-off between computational cost and solution quality. Based on these premises, it seems apparent that a direct computational comparison to previous algorithmic proposals would be questionable, if not arbitrary. Therefore, in our computational analysis we concentrate on the validation of our ranking methodology, which is the main focus of this paper. In fact, an appraisal of the merits and drawbacks of the many existing approaches to a priori routing in STD networks would be an interesting contribution, but this goes far beyond the scope of this paper.

To assess the quality and robustness of our methods we set up a particularly challenging experimental setting. To this aim, we concentrate on networks with a *grid* topology, and we exclude the *final steady state* adopted e.g. in Miller-Hooks and Mahmassani (2000) and Huang and Gao (2012). The combined impact of these two choices on the difficulty of the instances is discussed in detail in Section 4. For grids of different size and shape, we consider several different models for the *link behavior*, i.e., different width and shape of the fluctuations of link travel times and costs. The most important issue in our tests is that we address minimization of *costs*, in addition to minimization of travel times usually addressed in the literature. Instances involving costs instead of travel times turn out to be computationally much more demanding; a possible explanation of this behavior is given in Section 4. Nevertheless, our methods turn out to be reasonably stable under many different scenarios.

We remark that the restriction to grid networks fits the aims of our analysis, and cannot be considered as a limitation. In particular, our benchmark instances simulate road networks with congestion effects, and thus can be considered as a realistic representation (and most likely, a "worst case" example) of "real world" transportation networks. Benchmarks derived from road networks have been occasionally used in the literature on STD networks, but we believe that they would be redundant in our case. Besides, adapting the available network descriptions to our setting would be rather arbitrary, since there is no clearly established methodology for assessing the link behavior.

The paper is organized as follows. The definitions of discrete STD networks and of the related routing problems are given in Section 2. In Section 3 we provide our algorithms for the a priori shortest and K shortest path problems. In Section 4 we describe our test instances, and report computational results for finding the shortest and K shortest a priori paths. Finally, we summarize original contributions and directions for further research in Section 5. Appen-