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Finding a minimum cost path between a pair of nodes in a time-varying road network with a congestion charge

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ABSTRACT

The minimum cost path problem in a time-varying road network is a complicated problem. The paper proposes two heuristic methods to solve the minimum cost path problem between a pair of nodes with a time-varying road network and a congestion charge. The heuristic methods are compared with an alternative exact method using real traffic information. Also, the heuristic methods are tested in a benchmark dataset and a London road network dataset. The heuristic methods can achieve good solutions in a reasonable running time.

1. Introduction

There has been much research to provide solutions for solving vehicle routing and scheduling problems. However, most of the research published is based on models where the time between nodes on a road network is considered as fixed. In practice, this is not the case and the speed taken for any journey may vary significantly by the time of the day, the day of the week and the season of the year in which the journey takes place. For example, the traffic conditions at 1am are often different from those at 8am which is in the rush hour for commuters and as a result a journey starting at 8am may take a much longer time than the same one starting at 1am. The results from fixed speed models may produce schedules which lead to more vehicles spending time and fuels in congested traffic, which gives rise to further congestion and associated environmental costs. The fixed speed models may even lead to infeasible solutions for the practical problem. There are also economic and social costs due to missing delivery time windows and overtime costs when routes take longer time than planned.

In this paper, we propose two heuristic methods to determine the minimum cost path between a pair of nodes on a time-varying road network. Our cost structure includes three parts to the cost of the journey. One is the fuel cost which is influenced by the speed; the second is the driver cost which is related to the travelling time. The last is the congestion charge, when applicable. A congestion charge scheme (CCS) is a scheme of surcharging users of a transport network in periods of peak demand to reduce road congestion and decrease travelling times within the congestion charge zone. CCS helps to reduce pollution factors within the zone. CCS may be applied in a certain area during a certain time of the day. In general, some tolls may be collected on certain roads at certain times or at rates that change with time.

The implementation of CCS is an important factor when designing vehicle routing and scheduling systems. It can greatly affect the minimum cost paths on a real transportation network in a time-varying setting. This paper is motivated by the need for determining the cost minimizing paths on real size networks fast and with little computational effort as the existing algorithms are inefficient due to their CPU memory and/or computational time requirements. We test the performance of the proposed heuristic methods against an exact method to validate their applicability using real traffic information. The rest of the paper is organized as follows: the next section provides a literature review of previous work that utilizes time-varying (time-dependent) travel times. Section 3 describes the optimization problem of finding the minimum cost paths in a time-varying road network and presents the two heuristic methods. Section 4 investigates the performances of the heuristics on a benchmark dataset and compares them to the exact method devised in Chabini (1998). The following section presents a case study where we discuss the computational results obtained through the proposed heuristic methods on a real-life London data set. The last section presents conclusions and directions for further research.

2. Literature related to time-dependent travel time models for vehicle routing and scheduling

Ichoua, Gendreau, and Potvin (2003) give a brief literature review of the time-dependent vehicle routing problem (TDVRP).

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They point out that the TDVRP models proposed by Hill and Benton (1992) do not satisfy the "first-in-first-out" (FIFO) property as they represent the travel time as a step function of time. Ichoua et al. (2003) introduce a time-dependent model for the vehicle routing problem with time windows based on time-dependent travel speeds which satisfies the FIFO property. They implement a parallel tabu search approach and test its performance both in dynamic and static environments. The scheduling horizon is divided into three time intervals by taking into account the rush hours and three types of road are considered. The results show that the time-varying model provides significant improvements compared to the model with fixed travel times. Ichoua et al. (2003) also develop a dynamic vehicle routing model to adjust the vehicle routes that react to continuously changing traffic conditions in real time.

Eglese, Maden, and Slater (2006) show how the use of time-varying data can affect results for a hypothetical distribution operation and develop a model to use the historical data to construct a Road Timetable that shows the shortest time between nodes when the journeys start at different times. The shortest times and routes may vary as the speed of travel on individual roads may differ significantly by the hour of the day, by the day of the week and by the season of the year. The paper describes a case study using real speed data on a road network in the north of England.

Eglese and Black (2012) demonstrate the importance of speed with reference to vehicle routing. A route generated for optimizing distance may emit more CO_2 or other polluting gases due to slower speeds than a longer alternative route. So, reducing the travelling distance does not always reduce the CO_2 emissions.

Bektas and Laporte (2011) concern vehicle routing problems (VRPs) with different objective functions, but do not consider time-dependent travel times. They compare four different models with different objectives including distance minimizing, energy minimizing, weight load minimizing and cost minimizing objectives. They provide some numerical analyses on some small instances and conclude that minimizing the energy consumption is not equivalent to minimizing the cost. As the labour cost constitutes a major proportion of the total cost, the cost minimizing model focuses on the labour cost in order to reduce the total costs. Advances in engine technology lower the amount and cost of emissions hence lowering the overall total cost. Minimizing the cumulative weight load only does not necessarily imply energy minimization, particularly when time window restrictions are applied.

Cooke and Halsey (1966) present a dynamic programming algorithm for solving the all-to-one (from all nodes to one destination for any possible departure time) fastest path problem with time-dependent travel times over the discrete time horizon $(0,T]$. The algorithm is based on Bellman's optimality conditions for the shortest path problem (SPP) with time-dependent travel times. Based on the formulation proposed by Cooke and Halsey, Ziliaskopoulos and Mahmassani (1993) develop a label-correcting algorithm to solve the time-dependent SPP. Labels are stored in a vector, one for each time interval, and are maintained for every node and updated in a label-correcting fashion, i.e. the labels are upper bounds to the optimum path label until the algorithm terminates. The main characteristic of the algorithm is to scan all labels of a node for all possible departure times. A scan-eligible list is created to maintain all the nodes with the potential to improve at least one label of any node in the network. Note that Bellman's optimality principle is not satisfied on time-varying transportation networks when the objective is to determine the minimum cost path. In other words, real transportation networks do not satisfy the Cost Consistency property when waiting at the nodes is not allowed, i.e. leaving a node earlier does not necessarily cost less than leaving it later, in particular if a CCS is applied.

The stochastic dynamic network extension of the problem has been addressed by Hall (1986), Cheung (1998), Fu and Rilett (1998), Miller-Hooks and Mahmassani (1998, 2000, 2003), Pretolani (2000), and Huang and Gao (2012). These studies deal with finding the path with the shortest expected travel time on a dynamic network where the arc travel times are time-dependent random variables and the probability distributions vary with time. Multi-objective approaches have also been proposed within the context of hazardous materials transportation for finding the non-dominated paths by considering uncertain attributes such as travel time, population exposure, accident probability (e.g. Chang, Nozick, & Turnquist, 2005). Since this topic is beyond the scope of the paper, we refer the interested reader to Erkut, Tjandra, and Verter (2007) for a comprehensive review.

Although the literature on finding the shortest or fastest path is vast, there are few articles that attempt to determine the minimum cost path on a time-varying network environment. Pallottino and Scutella (1998) present an algorithmic paradigm, namely Chrono-SPT, for the dynamic shortest path problems using discrete models, i.e., they assume that the time varies in a discrete set. They analyze different implementation schemes by performing chronological type visits only on the non-redundant portion of an acyclic space–time network. Based on the reverse implementation of the Chrono-SPT and time-dependent SPP algorithm of Ziliaskopoulos and Mahmassani (1993), Miller-Hooks and Yang (2005) present reoptimization techniques to determine the updated fastest paths from all origins to a single destination when future travel times on the time-varying networks change. Their experimental results show that these techniques may provide substantial savings in the computational effort over determining the paths starting from scratch.

Chabini (1998) proposes an algorithm called the Decreasing Order of Time (DOT) algorithm to solve all-to-one fastest path problem and minimum cost problem in a time-varying road network by applying a backward labeling algorithm visiting the entire space–time network also using time discretization. The DOT algorithm has an exact computing complexity of $(SSP + nM + mM)$ where SSP is static shortest path, n is the number of nodes, m is the number of arcs and M is the number of time intervals. It compares the performance of DOT with three dynamic adaptations of label correcting algorithms using three types of data structures for node candidates list: the Deque data structure of Pape (1974) as described in Ziliaskopoulos and Mahmassani (1993), the 2-queue data structure of Gallo and Pallotino (1988) and the 3-queue data structure in Chabini (1998). The DOT algorithm has a better performance than the other three algorithms. However, both Chrono-SPT and DOT algorithms fail on real-size networks. We shall compare the performances of DOT and our heuristic methods using a real road network in a later section of the paper.

3. Finding the minimum cost path between two nodes

3.1. Preliminaries

We assume the driver will drive as fast as possible, subject to the speed of the traffic and any given maximum speed. So, the speed of the vehicle is always equal to the link speed in the dataset and is not a decision variable in the model.

The FIFO property means that if a vehicle leaves node i to go along arc (i,j) starting at time t, the time to arrive at node j for any other vehicle leaving node i and travelling along (i,j) after time t is later than the first vehicle. Provided that the FIFO property holds, Dijkstra's algorithm is able to find the optimal path between locations when the objective is to minimize the time. However, when the objective is minimum cost rather than shortest time