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The Multi-Handler Knapsack Problem under Uncertainty

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ABSTRACT

The Multi-Handler Knapsack Problem under Uncertainty is a new stochastic knapsack problem where, given a set of items, characterized by volume and random profit, and a set of potential handlers, we want to find a subset of items which maximizes the expected total profit. The item profit is given by the sum of a deterministic profit plus a stochastic profit due to the random handling costs of the handlers. On the contrary of other stochastic problems in the literature, the probability distribution of the stochastic profit is unknown. By using the asymptotic theory of extreme values, a deterministic approximation for the stochastic problem is derived. The accuracy of such a deterministic approximation is tested against the two-stage with fixed recourse formulation of the problem. Very promising results are obtained on a large set of instances in negligible computing time.

1. Introduction

The increasing competition due to both globalization of production processes and the movement of large quantities of freight between continents and countries creates the need for new tools for strategic and tactical decisions, that are able to deal with the stochastic nature of the processes involved. While this leads to new location and transportation problems (Tadei, Ricciardi, & Perboli, 2009; Tadei, Perboli, & Baldi, 2012), only a few studies deal with the stochastic study of packing problems (Perboli, Tadei, & Baldi, 2012). This is mainly due to the peculiarities of the literature related to packing. In fact, even if packing problems play a central role in transportation and logistics, the problems presented in the literature are mainly related to operational issues (Crainic, Perboli, & Tadei, 2012; Martello & Toth, 1979). Moreover, the parameter uncertainty affecting the final solutions such as the profit associated to item delivery or the cost of container renting is usually more evident in the planning phases rather than in the operational ones.

In this paper we introduce a new stochastic variant of the knapsack problem, the Multi-Handler Knapsack Problem under Uncertainty ($MHKP_u$). Given a set of items, characterized by volume and random profit, and a set of potential logistics handlers, the problem consists in finding a subset of items which maximizes the expected total profit. The profit is given by the sum of a deterministic profit and a stochastic profit oscillation, with unknown probability distribution, due to the random handling costs of the handlers.

A large number of real-life situations can be satisfactorily modeled as a $MHKP_u$, e.g. in financial and resource allocation. The general idea is to think of the capacity of the knapsack as the available amount of a resource (i.e. budget) and the items as activities to which this resource can be allocated (i.e. shares). Moreover, these items present profits which are random variables. The $MHKP_u$ may also appear as a subproblem of larger optimization problems.

A specific application of the $MHKP_u$ can be found in the automotive sector (Tadei, Perboli, & Della Croce, 2002). There the delivery of cars from manufacturers to dealers is not managed by the manufacturers themselves, but is delegated to specialized companies. These companies manage both the finishing operations on the cars (e.g. removal of the protective wax, installation of specific accessories, etc.) and the logistics operations linked to delivery to the dealers. In order to have a more flexible structure, the fleet of auto-carriers used to deliver the cars is only partially owned by each company, while a substantial part of the deliveries is sub-contracted to micro-companies with highly variable random costs. Moreover, the auto-carriers have different capacities due to the presence of specific technical features. From the point of view of the cars that must be delivered, the net profit for the company is affected by different factors, including delays in the finishing operations, additional costs due to violations of the negotiated deadlines or additional transportation costs.

Another example of real-world applications of the $MHKP_u$ comes from trans-continental naval shipping operations, where freight transportation from eastern ports to Europe and North America is managed by specialized companies. The competition between the transportation companies, as well as the possibility

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of managing the port cranes by different operators, force the companies to consider both the profit given by the shipped items and the additional costs due to the logistics operations.

The problem can be also seen as a relaxation of container loading problems where the capacity of the given containers are collapsed into one single container. This leads to an approximation of the real problem suitable in strategic and tactical planning, where the stochastic nature of the profits in more relevant than the actual loading of the items. Moreover, it is required to obtain accurate solutions within a limited computational effort in order to explore multiple scenarios of the underlying business model.

Other applications come from the domain of Smart City and City Logistics, and particularly in the last mile delivery. One of the trends is to substitute traditional single-echelon routing structures with two and three-echelon ones. The reason is the willingness to use "green" vehicles inside the city, whilst consolidating the freight in medium and small sized transshipment depots, called satellites (Crainic, Ricciardi, & Storchi, 2009; Crainic, Mancini, Perboli, & Tadei, 2010; Perboli, Tadei, & Vigo, 2011). In the satellites different sequences of consolidation operations are done by different workers. The different skill levels of the workers can cause delay in the operations reducing the overall profit. A similar problem is present in yard management, where the profit oscillations are given by the operations done by workers working for yard management companies, different in skills and reliability.

In general, the $MHKP_u$ arises in logistics and production scheduling applications, where a single item can be managed by several handlers (third-party logistics providers or sub-contractors), whose costs affect the net profit of the item itself. The large number of possible handler cost scenarios and the difficulty to measure the associated handler costs suggest the representation of these net profits as stochastic variables with unknown probability distribution.

This paper introduces the formulation of the stochastic problem. From this formulation, a deterministic approximation is derived. In particular, under a mild hypothesis on the unknown probability distribution, the deterministic approximation becomes a knapsack problem where the total expected profit of the loaded items is proportional to the logarithm of the total accessibility of those items to the set of handlers. Moreover, at optimality, the percentage of an item handled by any handler is given by a multinomial Logit model.

The paper is organized as follows. The literature review is introduced in Section 2. In Section 3 the model of the $MHKP_u$ is given. Section 4 is devoted to presenting the deterministic approximation of the $MHKP_u$, while in Section 5 its two-stage program with fixed recourse is given. Finally, in Section 6 the deterministic approximation and the two-stage program with fixed recourse are tested and compared on a set of newly introduced instances. The conclusion of our work is reported in Section 7.

2. Literature review

While different variants of the stochastic knapsack problem are present in the literature, the $MHKP_u$ is absent. For this reason, we will consider some relevant literature on similar problems, highlighting the main differences with the problem faced in this paper.

A first group of studies consider deterministic profits and random volumes, with the goal of maximizing the total expected value of selected items, while ensuring that the probability to satisfy the knapsack capacity is limited by some upper bounds. Usually, heavy assumptions on the distribution of the random volumes are considered (e.g. Kleinberg, Rabani, and Tardos (2000), Goel and Indyk (1999) where item volumes have a Bernoulli distribution, and Merzifonluoglu, Geunes, and Romeijn (2012), Barnhart and Cohn (1998) where the distribution is a Normal one). These assumptions on the distributions heavily limit the possibility to extend the results to other variants of the problem.

A second group of studies deals with random profits and the goal to assign a set of items to the knapsack in order to maximize the probability of achieving some target total value. They are usually more related to financial and economic issues than to the impact of the operations on the final revenue (Lisser & Lopez, 2010; Henig, 1990; Steinberg & Parks, 1979). Unfortunately, these problems differ from $MHKP_u$ because they consider the random profit associated only to the item, while in $MHKP_u$ the randomness is given by the interaction between the item and the handler managing the loading/unloading operations.

Finally, from a methodological point of view, the study most similar to the present paper is Perboli et al. (2012), where the authors consider the stochastic version of the Generalized Bin Packing Problem, a recently introduced packing problem where, given a set of bins characterized by volume and cost and a set of items characterized by volume and profit (which also depends on bins), a subset of items is selected for loading into a subset of bins which maximizes the total net profit, while satisfying the volume and bin availability constraints (Baldi, Crainic, Perboli, & Tadei, 2012). Similarly to $MHKP_u$, the item profits are random variables and the probability distribution of these random variables is assumed to be unknown.

3. The $MHKP_u$

In the $MHKP_u$ the item profits are random variables. In fact, they are composed by a deterministic profit plus a random term, which represents the profit oscillation due to the handling costs occurred by the different handlers for preparing items for loading. In practice, such profit oscillations randomly depend on the handling scenarios adopted by the handlers for preparing items for loading and are actually very difficult to be measured. This implies that the probability distribution of these random terms must be assumed as unknown.

Let it be

- I: set of items
- J: set of handlers
- L: set of handling scenarios for loading items into the knapsack
- p_i: non-negative deterministic profit of item i
- p_{ij}: non-negative deterministic profit of item i when loaded by handler j
- $\tilde{\theta}^{jl}$: random profit oscillation of any item when it is loaded by handler j under scenario $l \in L$
- $\tilde{p}_{ij}(\tilde{\theta}^{jl}) = p_{ij} + \tilde{\theta}^{jl}$: random profit of item i when loaded by handler j under scenario l
- y_i: boolean variable equal to 1 if item i is loaded, 0 otherwise
- x_{ij}: percentage of item i handled by handler j
- w_i: volume of item i
- W: knapsack capacity.

The $MHKP_u$ is formulated as follows

$$\max_{\{y,x\}} \quad \sum_{i\in I} p_i y_i + \mathbb{E}_{\{\tilde{\theta}^{jl}\}}\left[\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\tilde{p}_{ij}(\tilde{\theta}^{jl})x_{ij}\right] \tag{1}$$

subject to

$$\sum_{i\in I} w_i y_i \leqslant W \tag{2}$$

$$\sum_{j\in J} x_{ij} = y_i \quad i \in I \tag{3}$$

$$y_i \in \{0,1\} \quad i \in I \tag{4}$$

$$x_{ij} \geqslant 0 \quad i \in I, \quad j \in J. \tag{5}$$