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Measurement of multiperiod aggregative efficiency

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Abstract

This article proposes a new method for measuring an aggregative efficiency of multiple period production systems. Every organization or firm generates a time series of data that represent its performances in the resource utilization and output production over multiple periods, and often desires an aggregated measure of efficiency for several periods of interest. Data envelopment analysis (DEA) has become an accepted and well-known approach to evaluating efficiency performance in a wide range of cases. However, most of the DEA studies have dealt primarily with ways to gauge the efficiency of production in only a single period so this efficiency reflects the insufficient or partial performance of multiple period productions. The new method is developed through extensions of the concept of Debreu–Farrell technical efficiency and is applied to evaluating the efficiency of cable TV service units with 3-year data.

Keywords: Efficiency valuations; DEA; Time series data

1. Introduction

Data envelopment analysis (DEA), a nonparametric approach, has brought in possibilities for use in evaluating the efficiency performances of many different kinds of entities engaged in many different activities in many different contexts (e.g., Charnes et al., 1994; Cooper et al., 2000). Although great flexibility and extendibility exist, most of the DEA studies have dealt primary with cross sectional data and measured relative efficiencies in a single period, usually one year. Exceptions are window analysis in DEA (Charnes et al., 1985) and, under the umbrella of nonparametric approaches in econometric studies, Malmquist-type indexes of productivity (e.g., Caves et al., 1982; Fare and Grosskopf, 1996). Looking beyond the difference between their model details, we recocgnize that their common goal is to account for the changing patterns of efficiency performances over several periods of time. However, these approaches, while vital and practically useful, do not take into account an aggregated measure of efficiency for multiple period production systems.

Another exception is dynamic DEA. Nemoto and Goto (1999) proposed a dynamic DEA model to measure the overall efficiency of a multiperiod production system. This overall efficiency can be viewed as price or economic efficiency. They assumed perfect foresight with respect to the input costs over multiple periods and, within a usual production possibility set, determined an intertemporal efficient frontier in the way of minimizing the aggregated cost incurred by using inputs over time. However, even in a particular period, the assumption of exact costs of individual inputs is unrealistic (Thompson et al., 1990, 1995). Moreover, the true monetary value (or exact discount factor) of an input in the time horizon remains unknown in practice. Sueyoshi and Sekitani (2005) developed a method of how to measure returns to scale in the framework of the dynamic DEA of Nemoto and Goto (1999).

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Sengupta (1995, 1999) developed different types of dynamic DEA models in which various possible scenarios of aggregating input costs over time were considered. These models seek to determine the optimal levels of inputs (as the decision variables) over time. The computed optimal inputs are then used to determine the overall efficiency as the ratio of the two composite costs; one incurred by using the actual levels of inputs and the other evaluated at the optimal levels of inputs. Thus, as is similar to the Nemoto and Goto (1999) approach, it assumes that the future prices of inputs are available in determining the overall efficiency. However, Sengupta (1999) further extended his original ideas to incorporating the uncertainty of future input prices in the measurement of overall efficiency, while Nemoto and Goto (1999) assumed exact input prices.

We present a different attempt to measure the aggregative efficiency in the context of time serial data. To distinguish it from the previous work, we refer to the methodology as multiperiod data envelopment analysis (MDEA). This does not require any information on price data or preferential weightings of inputs and outputs over multiple periods, and yields a multiperiod aggregative efficiency (MAE) that corresponds conceptually to a technical (but not price or economic) efficiency of multiperiod production units. The development of MDEA is based on the concept of Debreu–Farrell's technical efficiency measurement. Following Debreu (1951), who provided the first measure of efficiency, Farrell (1957) proposed a nonparametric way of estimating technical efficiency, among others, on the bases of empirical input and output data. He suggested measuring the efficiency by means of comparing a target production unit with the unit on the efficient frontier. We extend this concept of efficiency measurement to multiperiod production units in order to arrive at MAE.

The paper is organized as follows. In the next section, we provide a motivating example and then extend the concept of the Farrell measurement to the context of time serial data for single input and multiple outputs. After this has been done we then put our ideas in a general and rigorously established form to accomplish MDEA. This is followed by an illustrative application to the 3-year data on cable TV service units. Finally, a summary and a sketch of further research opportunities conclude this paper.

2. Developments

2.1. Preliminaries

Table 1 shows a simple example where four decision making units (DMUs) produce different amounts of two outputs and consume the same unit amounts of single input in two periods $t = 1$, 2. Basically, using an ordinary DEA we can obtain the efficiency ratings of each DMU for individual periods. Listed in the last two columns, the results show that DMU_1 is efficient for both periods, DMU_2 is efficient for period $t = 1$ but inefficient for $t = 2$, and the other two DMUs are inefficient for both periods. These efficiency ratings obtained for individual periods are basically needed for efficiency valuations but reflect partial performances of multiperiod production units.

This brings into play an important question as to how we can achieve a multiperiod aggregative efficiency, shortly referred to as MAE. To carry out a panel data analysis, we assume underlying production technologies in which all of a period's input is expected to go into producing the output for the same period. We do not consider a special production system where current input amounts might be used to produce future period outputs. Specifically, the MAE is measured in a manner that a DMU's performance in a particular period is compared with the performance of all DMUs in the same period. The same way is actually taken in DEA to obtain the efficiency ratings in the last two columns of Table 1, but each of these ratings signifies a single period efficiency.

The desirable MAE measurement is expected to have the following basic and important features:

(a) It is straightforward to make a clear distinction between an always efficient DMU and a sometimes efficient DMU for all periods. For example, in Table 1 DMU_1 is efficient for both periods, while DMU_2 is efficient for the first period but inefficient for the second period. If only the two periods are of interest, then DMU_1 is always efficient but DMU_2 is sometimes efficient. Obviously, over the two periods, DMU_1 has no inefficiency (in terms of technical efficiency) while DMU_2 has inefficiency as 25% (= 1.25–1) of its output levels for the second period. In the DEA literature

Table 1
A simple example of time serial data

DMUs	Period $t = 1$			Period $t = 2$			Efficiency ratings in each period	
	Output1	Output2	Input	Output1	Output2	Input	$t = 1$	$t = 2$
1	3	5	1	5	3	1	1	1
2	5	3	1	4	2	1	1	1.25
3	2	4	1	2	2	1	1.25	1.5
4	2	2	1	2	2	1	2.0	1.5