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Decision Support

The discrete forward–reserve problem – Allocating space, selecting products, and area sizing in forward order picking

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ABSTRACT

To reduce labor-intensive and costly order picking activities, many distribution centers are subdivided into a forward area and a reserve (or bulk) area. The former is a small area where most popular stock keeping units (SKUs) can conveniently be picked, and the latter is applied for replenishing the forward area and storing SKUs that are not assigned to the forward area at all. Clearly, reducing SKUs stored in forward area enables a more compact forward area (with reduced picking effort) but requires a more frequent replenishment. To tackle this basic trade-off, different versions of forward–reserve problems determine the SKUs to be stored in forward area, the space allocated to each SKU, and the overall size of the forward area. As previous research mainly focuses on simplified problem versions (denoted as fluid models), where the forward area can continuously be subdivided, we investigate discrete forward–reserve problems. Important subproblems are defined and computation complexity is investigated. Furthermore, we experimentally analyze the model gaps between the different fluid models and their discrete counterparts.

1. Introduction

Picking highly-demanded stock keeping units (SKUs) directly from bulk storage typically requires removal from deep-lane pallet racks and causes unproductive travel between far-distant picking locations. Thus, especially in high-volume distribution, many warehouses are subdivided into a forward and a reserve area. The forward area serves as a "warehouse within a warehouse" and stores popular SKUs in easy-to-access racks, e.g., gravity flow racks, concentrated in a compact area. The use of a forward area improves order picking efficiency, but requires additional effort for replenishing the forward area from the reserve area (double handling). Clearly, reducing SKUs stored in forward area enables a more compact forward area (with reduced picking effort) but requires a more frequent replenishment. To tackle this basic trade-off, forward–reserve problems have been formulated and modeled to determine the SKUs to be stored in the forward area, the space allocated to each SKU, and the overall size of the forward area.

Hackman et al. (1990) were the first that formulated a mathematical model to allocate space in a continuously divisible forward area and proposed a greedy heuristic. Their work motivated the paper of Hackman and Platzman (1990) who proposed a generic discrete model for deciding which SKUs to pick from the automated (forward) area and how much space to allocate on which (forward) storage device to each selected SKU. They also developed a heuristic procedure with a good guaranteed performance whenever each allocation is a small fraction of storage space. Further contributions stem from Frazelle et al. (1994), who extended the model by regarding the size of a forward area as a decision variable, Van den Berg et al. (1998), who optimized unit-load replenishments that take place during busy and idle periods, Bartholdi and Hackman (2008), who analyzed two wide-spread real-world stocking strategies for small parts in a forward area, and Gu et al. (2010), who provided a branch-and-bound algorithm for solving the joint assignment and allocation problem. Except for the contribution of Hackman and Platzman (1990) and Van den Berg et al. (1998), all these studies presuppose the "fluid model", where a forward area can continuously be partitioned among SKUs. Clearly, this simplifying assumption might be justified if merely an approximate benchmark solution is sought. However, for a detailed stocking plan of the forward area, the fluid model shows some severe drawbacks (Bartholdi and Hackman, 2011):

- SKUs can only be stored in discrete units, so that a continuous distribution of space in either case can only be an approximation of reality. This approximation becomes the less accurate the larger an item compared to the size of the shelves.
- Often the number of units stocked in forward area cannot be increased piece-wise but only in steps of multiple units. For instance, consider a gravity flow rack where each lane is filled

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with units of a single homogeneous product. In this case, e.g., either 0, 1000 or 2000 units can be stored depending on whether none, one or two lanes (each having a capacity for 1000 units) are allocated to the respective SKU.

- Finally, some SKUs show sub-additivity with regard to space because they can be stored in a nested fashion. Consider bathtubs where two units occupy only a little more space than a single unit. This coherency cannot be considered within the continuous forward–reserve problem, because here a linear increase of space utilization is presupposed.

To avoid these shortcomings, this paper considers the discrete forward–reserve problem. Specifically, we investigate three different problem settings each within two important parameter constellations, so that in total six problem cases are treated. The most basic problem setting is the discrete forward–reserve allocation problem (DFRAP), where the given space of a forward area is to be partitioned among a predetermined set of SKUs (see Section 2). The discrete forward–reserve assignment and allocation problem (DFRAAP) combines the space allocation problem with the assignment problem of selecting the products to be stored in the forward area (see Section 3). Finally, the discrete forward–reserve allocation and sizing problem (DFRASP) treats the allocation problem jointly with the sizing problem, i.e., for a given set of products a forward area of variable size is to be allocated (see Section 4).

With regard to the parameter constellations, we differentiate between variable storage modes and equally sized shelves. In the former case, for any product different storage alternatives exist. For instance, SKUs can be stored in small or large boxes, on one or more pallets in shelves of variable size, or in one or more lanes of varying capacity in a gravity flow rack, where each alternative corresponds to a certain storage mode. Each mode is defined by a specific space utilization and the corresponding discrete number of stored product units. Clearly, the confinement to equally sized shelves defines a special case of variable storage modes. Here, the storage mode of each SKU is already defined and the decision reduces to determining the discrete number of equally sized shelves (each storing an identical multiple of units) to be assigned to any product. Typically, standardized racks with equally sized shelves, e.g., identical lanes in a gravity flow rack, are applied in a wide range of distribution centers as they allow for fast and simple re-assignments, i.e., some products that have been stored forward during the last period will be partly or fully replaced by new products within the next period. Furthermore, standardized racks, typically, cause less investment cost.

In the following, the DFRAP, the DFRAAP, and the DFRASP are treated in Sections 2, 3, and 4, respectively. In each section both parameter constellations are investigated by defining the respective problem version and either providing a polynomial time solution procedure or proving $\mathcal{NP}$-hardness. Furthermore, in each case we explore the loss in precision of the continuous problem version compared to the discrete case in a comprehensive computational study. Finally, Section 5 concludes the paper.

2. The discrete forward–reserve allocation model (DFRAP)

2.1. DFRAP with variable storage modes

Consider a predefined set P of products (or SKUs) to be stored in a forward area of given size S, where due to the compact size of the forward area the locations of SKUs are assumed to not affect picking efficiency. Each product $i \in P$ can be stored in one of n_i possible modes $j = 1, \ldots, n_i$ in the forward area. Associated with each mode j for SKU i is a value $a_{ij} > 0$ which gives the (integral) number of units of SKU i that can be stored in mode j. The space required by storing product i in mode j is denoted by w_{ij}. Throughout the paper we assume that the storage modes of each product i are labeled so that $w_{i1} < w_{i2} < \cdots < w_{in_i}$. According to this, we consider only non-dominated storage modes for each SKU $i \in P$, i.e., $a_{i1} < \cdots < a_{in_i}$.

As formulated in the mathematical model (1)–(4), the DFRAP decides on the mode in which each SKU is stored in the forward area (restriction (2)). Equivalently, it decides on the number of units to be stored per product without exceeding given storage space S (restriction (3)) which is assumed to be at least as large as $\sum_{i \in P} w_{i1}$. Otherwise, not all products of the predefined set could be stored in the forward area. Binary variables x_{ij} defined in (4) indicate which storage mode is chosen for SKU i. Clearly, the more space is associated with each SKU the less restocks are required per time unit. If d_i represents the total demand for product i during the planning period, e.g., a year, then the number of restocks per SKU can simply be calculated by dividing the total demand d_i by the number of units stored in the forward area of the respective product. Note that the underlying assumptions with regard to restocks are discussed in detail by Bartholdi and Hackman (2008).

When weighting the number of restocks with product specific replenishment cost c_i the objective (1) of the DFRAP is to minimize the overall restock cost per planning period:

$$\text{DFRAP : Minimize} \quad C_1(x) = \sum_{i \in P} \sum_{j=1}^{n_i} c_i \frac{d_i}{a_{ij}} x_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{j=1}^{n_i} x_{ij} = 1 \quad \forall\, i \in P \tag{2}$$

$$\sum_{i \in P} \sum_{j=1}^{n_i} w_{ij} x_{ij} \leqslant S \tag{3}$$

$$x_{ij} \in \{0, 1\} \quad \forall\, i \in P;\ j = 1, \ldots, n_i \tag{4}$$

The DFRAP is mathematically equivalent to the well-known multiple-choice knapsack problem (MCKP), which becomes obvious when interpreting the parameters of the MCKP as follows:

- SKUs correspond to classes and (storage) modes (of a SKU) correspond to items (of a class),
- the profit of item j of class i is $p_{ij} = -c_i d_i / a_{ij}$ (minimizing $\sum_i \sum_j p_{ij} x_{ij}$ is equivalent to maximizing $\sum_i \sum_j - p_{ij} x_{ij}$),
- the weight of item j of class i equals the space w_{ij} required by storing SKU i in mode j,
- the capacity of the knapsack is $c = S$,
- the size of class i is n_i.

Clearly, the DFRAP is $\mathcal{NP}$-hard as the MCKP, which is a generalization of the knapsack problem, is well-known to be $\mathcal{NP}$-hard (see Kellerer et al., 2004, Chap. 11). Furthermore, pseudo-polynomial solution procedures for the MCKP exist, e.g., the dynamic programming approach of Dudzinski and Walukiewicz (1987), so that it follows that the DFRAP is weakly $\mathcal{NP}$-hard.

2.2. DFRAP with equally sized shelves

For adopting the basic DFRAP to the case of equally sized shelves of a rack or lanes of identical capacity in a gravity flow rack, we keep to the notation introduced in Section 2.1 with a slightly different interpretation of S, w_{ij}, and a_{ij}. Parameter S denotes the total (integral) number of shelves that are available for storing the products of set P in the forward area. Hence, we let w_{ij} denote the number of shelves that are available when storing SKU i in mode j. As we agreed on allowing any integral number of shelves between 1 and n_i for forward storing of SKU i, where $n_i = S - |P| + 1$