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Decision Support

Reliability analysis and optimization of weighted voting systems with continuous states input

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Abstract

Weighted voting systems are widely used in many practical fields such as target detection, human organization, pattern recognition, etc. In this paper, a new model for weighted voting systems with continuous state inputs is formulated. We derive the analytical expression for the reliability of the entire system under certain distribution assumptions. A more general Monte Carlo algorithm is also given to numerically analyze the model and evaluate the reliability. This paper further proposes a reliability optimization problem of weighted voting systems under cost constraints. A genetic algorithm is introduced and applied as the optimization technique for the model formulated. A numerical example is then presented to illustrate the ideas.

Keywords: Weighted voting systems (WVS); Reliability analysis; Reliability optimization; Genetic Algorithms (GA)

1. Introduction

Weighted voting systems (WVS) have attracted a lot of attention recently (see, e.g. Latif-Shabgahi et al., 2004; Levitin, 2001, 2002a,b, 2003, 2004, 2005a; Parhami, 1994; Xie and Pham, 2005; Yacoub, 2003) as such systems are widely used in pattern recognition, human organization systems and technical decision making systems. They are a generalization of traditional k-out-of-n systems, with the following properties:

1. Each voting unit makes individual independent decision.
2. Each voting unit has its weight.
3. The decision of the system is based on the information from the individual voting units of the system.

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Nomenclature

Acronyms

WVS	weighted voting system
WVC	weighted voting classifier
GA	genetic algorithms
MC	Monte Carlo
AM	analytical method
UMGF	universal moment generating function

Notations

N	number of units belonging to WVS
w_i	weight of unit i
X	continuous input of the entire system
Y_i	output of individual voting unit i
Y	output of entire voting system
$g_x(y)$	probability density function of system output Y given the input $X = x$
$g_x^i(y_i)$	probability density function of output Yi given the input $X = x$
$p(x)$	probability that the decision of entire system is correct given $X = x$
R	reliability of entire system given any input
$f(x)$	probability density function of input X
a	threshold of the entire system for judging if the output is correct
M	number of different types of voting units
V_{im}	structure of the voting system, $V_{im} = 1$ if unit of type m allocated in position i of the voting system
J_m	number of voting units of type m
C_m	cost of voting units of type m
c_i	cost of voting unit at position i in the entire system
C	cost limit for entire system
σ_m^R	standard deviation of the distribution of the output of voting units of type m in resource R
σ_i^S	standard deviation of the distribution of the output of voting unit at position i in the entire voting system

The individual units in weighted voting systems and their corresponding outputs are subject to different errors, which are defined into three types (Levitin and Lisnianski, 2001). The system incorporates all the unit decisions into one unanimous system output D. In a system with discrete input, given input being either 1 (proposition to be accepted) or 0 (proposition to be rejected), D is based on a weighted sum of the units voting D. If this sum exceeds a particular threshold value, the system output is 1, otherwise it is zero. The system fails when such an output can not be generated by the rules. In this way, the entire weighted voting system reliability is defined as the probability that the system can successfully vote a correct output, which depends on the unit weights and the system threshold (Levitin and Lisnianski, 2001).

Reliability estimation of the weighted voting systems is a complex problem, which has attracted attention from many researchers. Nordmann and Pham (1998) first proposed the formula for calculating reliability of a WVS which is simplified by two given restrictions. However, the computational complexity increases exponentially in the number of units. Xie and Pham (2005) propose a simpler method to calculate the WVS reliability and saddle point approximation techniques are applied to simplify the calculation. In a series of papers Levitin (2001–2005a) evaluates the reliability function based on the universal z-transform (or universal moment generating function, UMGF) technique, which is proven to be a very effective method for numerical implementation of obtaining the reliability of the multi-state weighted voting system (Levitin, 2005b; Levitin et al., 2007).