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Specification and estimation of primal production models ☆

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ABSTRACT

While estimating production technology in a primal framework production function, input and output distance functions and input requirement functions are widely used in the empirical literature. This paper shows that these popular primal based models are algebraically equivalent in the sense that they can be derived from the same underlying transformation (production possibility) function. By assuming that producers maximize profit, we show that in all cases, except one, the use of ordinary least squares (OLS) gives inconsistent estimates irrespective of whether the production, input distance and input requirement functions are used. Based on several specifications of the production and input distance function models, we conclude that one can estimate the input elasticities and returns to scale consistently using instruments on only one regressor. No instruments are needed if either it is assumed that producers know the technology entirely (including the so-called error term) or a system approach is used. We used Norwegian timber harvesting data to illustrate workings of various model specifications.

1. Introduction

Specification and estimation of the production function is important in production economics. In spite of many advances in the last 80 plus years since the introduction of Cobb–Douglas production function (Cobb and Douglas, 1928) in 1928 some of the fundamental issues are still debated. The two main issues of concern are specification and estimation of the underlying technology. The specification issue is important because there are many different ways in which one can specify the underlying technology. Although these alternative specifications are algebraically the same, they are not the same from econometric estimation point of view. These specifications use different econometric assumptions, and their data requirements are often different. Needless to say that the empirical results differ and this creates a big problem to the applied researchers who want to know which approach is appropriate to use. The choice is often dictated by what is endogenous (choice/decision variables) to the producers, and what is the objective of the producers. Cost minimization and profit maximization behaviors are widely discussed in microeconomics. Here we assume that the producers maximize profit in deciding their optimal output and input quantities (which are endogenous). Although the endogeneity issue was first addressed in Marschak and Andrews in 1944, it is still debated whether it is necessary to use a system approach to handle endogeneity problems.[1] In this paper we address these issues both theoretically and empirically. Our focus is on the primal specifications. We address the endogeneity issue primarily from economic theory (producer behavior) point of view as in Hoch (1958, 1962), Mundlak and Hoch (1965), Mundlak (1961) and Zellner et al. (1966). Furthermore, we discuss both single and system approaches for estimating the technology using cross-sectional data under a variety of cases.

Since the endogeneity problem comes from what are decision variables to the producers and what is the objective (economic behavior) of the producers, it is likely that one method cannot handle all situations. Zellner et al. (1966) showed that if producers maximize expected profit, the use of OLS in estimating the production function representation of the technology is appropriate in the sense that the OLS estimators are consistent. However, if producers know the so-called ‘unobserved’ (to the researchers) managerial input or management (Mundlak, 1961), the OLS estimators of the production function will be inconsistent even under expected profit maximization behavioral assumption. Similarly, if producers minimize cost and output is exogenously given, the use of OLS to the input distance function (not the production function) is appropriate in the sense that the OLS estimators are consistent. In a multiple output case (not considered here), the use of OLS to the output distance function formulation is appropriate under the

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[1] See Nerlove (1965) for a comprehensive treatment of the issue from a system approach. Recently Levinsohn and Petrin (2003) addressed the issue from a panel data point of view. They proposed use of a single equation approach in which investment is used as a control for correlation between input levels and firm-specific effects.

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assumption that producers maximize revenue and inputs are exogenously given (Coelli, 2000). Kumbhakar (2011) show that the use of OLS to either input or output distance function will give inconsistent parameter estimates if producers maximize return to the outlay (Färe et al., 2002) and both inputs and outputs are endogenous (choice variable). The use of OLS in the input requirement function formulation (Diewert, 1974) gives inconsistent parameter estimates if there are more than one endogenous input variables. This is the case even if outputs are exogenous. Various formulations of the underlying production technology are routinely used in the literature without discussing the endogeneity issue that we focus here.[2]

The rest of the paper is organized as follows. Section 2 introduces both the Cobb–Douglas and translog transformation functions and show how one can derive the production, input distance and input requirement functions by using different normalizations. Section 3 deals with estimation of the Cobb–Douglas and translog specifications in many different forms using a single equation framework. We do the same in Section 4 using the system approach. Section 5 describes the data and Section 6 reports results from both Cobb–Douglas and translog models from both the single and system approaches. Section 7 concludes the paper.

2. Representations of the transformation function

2.1. Cobb–Douglas transformation function

Assume that a producer uses J inputs x to produce a single output y. The functional relationship between x and y is usually described by a production function $f:\mathbb{R}^K_+ \to \mathbb{R}_+$ where $y = A\, f(x)$, where A is the efficiency parameter (function). We write the relationship in a more general form, viz., $Af(y,x) = 1$, and call it a transformation function instead of a production function which becomes a special case. Suppose that $f(\cdot)$ is Cobb–Douglas (CD) so that we can write it as

$$\textbf{CD transformation function}:\ Ay^{\alpha}\prod_j x_j^{\beta_j} = 1 \tag{1}$$

We now show that various primal functions representing the above transformation function can be derived simply by using different identifying (normalizing) constraints. Note that for the CD specification in (1) we need to normalize one parameter.

If we normalize $\alpha = -1$ then we get the standard CD production function specification, viz.,

$$\textbf{Production function}:\ y = A\prod_j x_j^{\beta_j} \tag{2}$$

If we rewrite (1) as

$$Ax_1^{\sum_j \beta_j} y^{\alpha}\prod_{j=2}\{x_j/x_1\}^{\beta_j} = 1 \tag{3}$$

and use the normalization $\sum_j\beta_j = -1$, then we get the input distance function (IDF) formulation (Shephard, 1953, 1970), viz.,

$$\textbf{Input distance function}:\ x_1 = Ay^{\alpha}\prod_{j=2}\{x_j/x_1\}^{\beta_j} \tag{4}$$

Finally, if we normalize $\beta_1 = -1$ in (1) it can be rewritten as

$$\textbf{Input requirement function}:\ x_1 = Ay^{\alpha}\prod_{j=2} x_j^{\beta_j} \tag{5}$$

which is the input requirement function (IRF) introduced by Diewert (1974).

It is clear from the above that starting from the transformation function in (1) one can obtain the production function, the input distance function, and the input requirement function simply by using different normalizations. No additional assumptions are necessary for this. Theoretically, the transformation function in (1) can be traced back starting from any of these functions. Thus, the production function, input distance function and the input requirement function are algebraically equivalent in the sense that they are all derived from the same transformation function but use different normalizations. This is true for flexible functional forms such as the translog which is shown next.

2.2. Translog transformation function

As before we assume that a producer uses a J inputs x to produce a single output y. The functional relationship between x and y is expressed as $Af(y,x) = 1$, where $f(y,x)$ is assumed to be translog (TL), i.e.,

TL transformation function:

$$\ln f(y,x) = \alpha_y \ln y + \frac{1}{2}\alpha_{yy}\ln y^2 + \sum_j \beta_j \ln x_j + \frac{1}{2}\sum_j\sum_k \beta_{jk}\ln x_j \ln x_k + \sum_j \delta_{jy}\ln x_j \ln y \tag{6}$$

where $\beta_{jk} = \beta_{kj}$. Note that we need to impose $(J+2)$ identifying/normalizing constraints for the model in (6). If one uses the following normalizations $\alpha_y = -1$, $\alpha_{yy} = 0$, $\delta_{jy} = 0\,\forall j = 1,\ldots,J$ in (6) the standard translog production function is obtained, which is

TL production function:

$$\ln y = \alpha_0 + \sum_j \beta_j \ln x_j + \frac{1}{2}\sum_j\sum_k \beta_{jk}\ln x_j \ln x_k + u \tag{7}$$

where $\ln A = \alpha_0 + u$.

If we rewrite (6) as

$$\begin{aligned}\ln f(y,x) &= \alpha_y \ln y + \frac{1}{2}\alpha_{yy}\ln y^2 + \sum_j \beta_j \ln(x_j/x_1) \\ &+ \frac{1}{2}\sum_j\sum_k \beta_{jk}\ln(x_j/x_1)\ln(x_k/x_1) + \sum_j \delta_{jy}\ln(x_j/x_1)\ln y \\ &+ \left[\sum_j \beta_j\right]\ln x_1 + \left[\sum_j\left\{\sum_k \beta_{jk}\right\}\ln x_j\right]\ln x_1 \\ &+ \left[\sum_j \delta_{jy}\right]\ln y \ln x_1\end{aligned}$$

and use the following normalizations $\sum_j\beta_j = -1$, $\sum_j\beta_{jk} = 0, \forall k$, $\sum_j\delta_{jy} = 0$ the input distance function representation is obtained, which is,

TL IDF:

$$\ln x_1 = \alpha_0 + \sum_{j=2}\beta_j \ln\hat{x}_j + \frac{1}{2}\sum_{j=2}\sum_{k=2}\beta_{jk}\ln\hat{x}_j\ln\hat{x}_k + \alpha_y \ln y + \frac{1}{2}\alpha_{yy}\ln y^2 + \sum_{j=2}\delta_{jy}\ln\hat{x}_j \ln y + u, \tag{8}$$

where $\hat{x}_j = x_j/x_1,\ j = 2,\ldots,J$.

Finally, if we use the following normalizations in (6) $\beta_1 = -1$, $\beta_{1j} = 0\,\forall j$, $\delta_{1y} = 0$, the input requirement function is obtained, which can be written as

[2] There are too many papers which use various specifications we refer to in this paper in the next section. Some of the recent papers in the operation research literature are: Boussemart et al. (2009), Parelman and Santín (2009), among many others.