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Decision Support

A multiple criteria sorting method where each category is characterized by several reference actions: The Electre Tri-nC method

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ABSTRACT

This paper presents Electre Tri-nC, a new sorting method which takes into account several reference actions for characterizing each category. This new method gives a particular freedom to the decision maker in the co-construction decision aiding process with the analyst to characterize the set of categories, while there is no constraint for introducing only one reference action as typical of each category like in Electre Tri-C (Almeida-Dias et al., 2010). As in such a sorting method, this new sorting method is composed of two joint rules. Electre Tri-nC also fulfills a certain number of natural requirements. Additional results on the behavior of the new method are also provided in this paper, namely the ones with respect to the addition or removal of the reference actions used for characterizing a certain category. A numerical example illustrates the manner in which Electre Tri-nC can be used by a decision maker. A comparison with some related sorting procedures is presented and it allows to conclude that the new method is appropriate to deal with sorting problems.

1. Introduction

Sorting problems arise in several real-life activities. In this paper, we are interested in *decision aiding contexts* in which the objects of a decision (actions, alternatives, etc.) must be sorted, or assigned to a set of categories. Such an assignment is based on the evaluation of each action according to multiple criteria. As we have shown in Electre Tri-C framework (Almeida-Dias et al., 2010), there are different manners to deal with the sorting approach. In what follows, we shall present a new approach, in which three assumptions are also taken into account, where only the Assumption 3 differs from the ones in Electre Tri-C.

Assumption 1. The set of categories to which the actions must be assigned to is completely ordered (from the worst to the best, from the lowest priority to the highest priority, from the most risky to the least risky, from the least consensual to the most consensual, and so on). In general, numbering the categories from 1 to q must be coherent with respect to the increasing preferences on the criteria.

Assumption 2. Each category is conceived *a priori* to receive actions, which will be or might be processed in the same way (at least in the step that follows the assignment).

Assumption 3. Each category is characterized by a subset of reference actions judged by the decision maker as representative, or informative of the actions that should be assigned to such a category. The *decision maker* is able, through a co-construction interactive process with the *analyst*, to provide the performances of the reference actions for characterizing each category according to Assumption 2.

The new method proposed in this paper, called Electre Tri-nC, takes appropriately into account the Assumption 3. As in Electre Tri-C, the objective of Electre Tri-nC is not to discover a pre-existing set of categories where the studied actions would naturally be assigned to. The objective is rather *to help* decision makers to characterize an appropriate set of categories to receive actions according to Assumption 2. This means that categories must be conceived according to the way actions will be or might be processed in the step that follows the assignment. In other words, the reason that leads to group actions in categories comes from the fact that the actions assigned to a given category will be or might be processed in the same way in the following step. We would like to call the attention of the reader to the following aspects. In several concrete decision aiding situations (see, for instance, the examples in Almeida-Dias et al. (2010, Section 1) and

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the numerical example in Section 5), it is not possible to refer to an objective set of reference actions defined *a priori*, which can be used as a starting point for characterizing the set of categories according to Assumption 2. The reference actions should express a willing of the decision maker for characterizing the categories.

The characterization of the categories based on representative reference actions gives a "fuzzy" position with respect to the frontier between a given category and the two consecutive adjacent ones. Some of such characteristic reference actions can be incomparable or even indifferent to some actions to be assigned to the categories. These two reasons seem enough to justify that a given action can be assigned to more than one consecutive categories. Thus, Electre Tri-nC has been conceived to be able to propose to the decision maker all the possible assignments according to Assumption 2.

The rest of this paper is organized as follows. Section 2 is devoted to the problem statement framework. Section 3 presents the assignment procedure, the foundations of Electre Tri-nC, by putting into light new possibilities of such a sorting method. Section 4 introduces the properties of Electre Tri-nC as well as the impact of a new characterization of the set of categories. Section 5 provides a numerical example. Section 6 includes a comparison to other sorting methods, where characteristic reference actions are initially used for representing the willing of the decision maker about the assignment to each category. Finally, the last section offers our concluding remarks and some avenues for future research.

2. Problem statement

This section is devoted to the main concepts, definitions, and notation as well as the structural requirements concerning the Electre Tri-nC method.

2.1. Concepts, definitions, and notation

Let $A = \{a_1, a_2, \ldots, a_i, \ldots,\}$ denote the set of *potential actions*. This set of actions can be completely known *a priori* or it may appear progressively during the decision aiding process. The objective is to assign these actions to a set of *completely ordered categories*, denoted $C = \{C_1, C_2, \ldots, C_h, \ldots, C_q\}$, with $q \geqslant 2$ (with $q = 1$, there is no sorting problem). Suppose that a *coherent set of n criteria*, denoted $F = \{g_1, g_2, \ldots, g_j, \ldots, g_n\}$, with $n \geqslant 3$, has been defined in order to evaluate any action considered to be assigned to a certain category (see Roy, 1996). Let us notice that if $n < 3$, then the concept of concordance is not really pertinent. For such a reason, when using Electre family of methods, it is recommended to have at least three criteria.

Each criterion g_j will be considered as a *pseudo-criterion*, which means that two thresholds are associated to g_j: an *indifference threshold*, q_j, and a *preference threshold*, p_j, such that $p_j \geqslant q_j \geqslant 0$. These thresholds are introduced in order to take into account the imperfect character of the data from the computation of the performances $g_j(a)$, for all $a \in A$, as well as the arbitrariness that affects the definition of the criteria. For more details about the definition of such thresholds, see, for instance, Almeida-Dias et al. (2010, Section 2). Let us notice that the case where $p_j = q_j = 0$, for all $g_j \in F$, is not excluded, but such a case must be considered as a very particular realistic case. In what follows, assume, without loss of generality, that all criteria $g_j \in F$ are to be maximized, which means that the preferences increase when the criteria performances increase too.

When using the outranking concept, the main idea is that "a outranks a'" according to g_j, denoted aS_ja', if "a is at least as good as a'" on criterion g_j. Due to the definition of the indifference thresholds, q_j, it is quite natural to consider that such an assertion is validated, without ambiguity, when $g_j(a) - g_j(a') \geqslant -q_j$. But, when $-p_j \leqslant g_j(a) - g_j(a') < -q_j$, the possibility of indifference between a and a' cannot be excluded. This indifference is less and less credible when $g_j(a) - g_j(a')$ moves closer to $-p_j$.

Let $\sigma(a,a')$ denote the credibility of the comprehensive outranking of a over a', which reflects the strength of the statement "a outranks a'" (denoted aSa') when taking all the criteria from F into account. This aggregation issue is based on a *single vector of weights*, denoted w_j, such that $w_j > 0, j = 1, \ldots, n$, which is associated to the set of criteria. Additionally, a vector of *veto thresholds*, denoted v_j, such that $v_j \geqslant p_j$ can also be associated to the set of criteria. For more details on the computation of $\sigma(a,a')$, see, for instance, Almeida-Dias et al. (2010, Section 2).

Let us introduce now the set of reference actions. Let $B_h = \{b_h^r, r = 1, \ldots, m_h\}$ denote a subset of *reference actions* introduced to characterize category C_h, such that $m_h \geqslant 1$ and $h = 1, \ldots, q$. Notice that C_1 is the worst category and C_q is the best one, with $q \geqslant 2$. Let $B \cup \{B_0, B_{q+1}\}$ denote the set of $(q + 2)$ subsets of reference actions, or the set of all reference actions, such that $B = \{B_1, B_2, \ldots, B_h, \ldots, B_q\}$. The two particular subsets of reference actions, denoted $B_0 = \left\{b_0^1\right\}$ and $B_{q+1} = \left\{b_{q+1}^1\right\}$, contains two reference actions defined as follows: $g_j\left(b_0^1\right)$ is the worst possible performance on criterion g_j, and $g_j\left(b_{q+1}^1\right)$ is the best possible performance on the same criterion g_j, for all $g_j \in F$. The worst and the best possible performances must be chosen such that, for any action a, one has $g_j(b_0^1) < g_j(a) < g_j\left(b_{q+1}^1\right)$, for all $g_j \in F$. Moreover, for all $g_j \in F$, one has $g_j(b_1^r) - g_j\left(b_0^1\right) > 0, r = 1, \ldots, m_1$, and $g_j\left(b_{q+1}^1\right) - g_j\left(b_q^s\right) > 0, s = 1, \ldots, m_q$.

The comparison of an action a to the characteristic reference actions $b_h^r, r = 1, \ldots, m_h$, provides m_h credibility indices of each type, $\sigma(a, b_h^r)$ and $\sigma(b_h^r, a)$. In order to make a judgment regarding the way in which an action a is placed with respect to the category C_h, it is suitable to find an aggregation operator that allows to obtain a representative credibility index for each action a with respect to each subset of reference actions, B_h, $h = 1, \ldots, q$. As for the case of decision aiding sorting methods using a set of unordered categories (see, for instance, Perny, 1998; Henriet, 2000; Belacel, 2000; Léger and Martel, 2002), the *max* operator is also a natural choice in our framework as follows:

Definition 1 (*Categorical credibility indices*).

(a) $\sigma(\{a\}, B_h) = \max_{r=1,\ldots,m_h}\{\sigma(a, b_h^r)\}$.
(b) $\sigma(B_h, \{a\}) = \max_{s=1,\ldots,m_h}\{\sigma(b_h^s, a)\}$.

The credibility indices computed according to Definition 1(a) can be interpreted as the *categorical outranking degrees* of action a over the subset of reference actions B_h. Similarly, the credibility indices computed according to Definition 1(b) can be interpreted as the *categorical outranked degrees* of action a over the subset of reference actions B_h.

The justification for these two interpretations are as follows. The categorical credibility indices, $\sigma(\{a\},B_h)$ and $\sigma(B_h,\{a\})$, are used for managing the assignment process (see Section 3.1). Thus:

(1) When defining the outranking credibility of an action a over a subset of reference actions B_h, $\sigma(\{a\},B_h)$, it seems natural to impose *a priori* that such a credibility degree should verify the following two axioms:

Axiom 1. If $B_h = \left\{b_h^1\right\}$, then, for any action a, $\sigma(\{a\}, B_h) = \sigma\left(a, b_h^1\right)$.