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Decision Support

Optimal dynamic pricing of inventories with stochastic demand and discounted criterion

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ABSTRACT

We consider a continuous time dynamic pricing problem for selling a given number of items over a finite or infinite time horizon. The demand is price sensitive and follows a non-homogeneous Poisson process. We formulate this problem as to maximize the expected discounted revenue and obtain the structural properties of the optimal revenue function and optimal price policy by the Hamilton–Jacobi–Bellman (HJB) equation. Moreover, we study the impact of the discount rate on the optimal revenue function and the optimal price. Further, we extend the problem to the case with discounting and time-varying demand, the infinite time horizon problem. Numerical examples are used to illustrate our analytical results.

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1. Introduction

Many firms face the problem of selling a fixed stock of items over a finite horizon or infinite horizon. For examples, airlines sell a number of seats before planes depart, hotels sell rooms before midnight, theaters sells seats before a specified time, and retailers sell seasonal products. These cases are all in a finite time horizon. On the other hand, most of the consumer products need to be sold in a finite or infinite time horizon. These problems have been investigated extensively in the literatures. Kimes [14] gives a general overview on investigating hotel industry. Bitran and Gilbert [3], Liberman and Yechiali [17] and Rogers and Williams [18] develop corresponding models for hotel management. Lee and Hersh [16] use the discrete time dynamic programming to develop optimal rules when demands are modeled as stochastic processes. Zhao and Zheng [19] investigate the optimal dynamic pricing for perishable assets with non-homogeneous demand. Bülüt et al. [5] consider a retailer selling a fixed inventory of two perishable products over a finite horizon. All these papers are based on the expected revenue maximizing. There are two criteria in order to analyze a supply chain. One has the objective function of maximizing the expected revenue and another one has the objection function of maximize the expected discounted revenue. An example is that estimating the net present value of a firm as a discounted sum of its future cash flows, based on the current state of its operations (see Bertsekas [1] and Fleming and Soner [11]). The former investigates the discrete time dynamic programming problem and the latter analyzes the problem with continuous time. The problem with discounting has not been studied before. In this paper, we study the problem based on the second criterion.

We consider a continuous time dynamic pricing problem for selling a given number of items over finite or infinite horizons. For the finite time horizon, given number of items to be sold in the horizon, demand is assumed to follow a non-homogeneous Poisson process and the demand intensity is determined by the price offered by the firm. Therefore, the firm can control the demand intensity by adjusting the price it offers. Items unsold at the end of the time horizon are disposed at a given salvage value, which we can assume to be zero without loss of generality. Inventory is not replenishable, and unsatisfied demand is lost without the penalty cost. The firm's objective is to maximize the expected discount revenue by using a dynamic pricing policy. We then extend the finite horizon case to the infinite horizon case.

Dynamic pricing problem is a well studied problem in revenue management. For a detailed review, the reader is referred to Elmaghraby and Keskinocak [7] and Bitran and Caldentey [2]. Gallego and Van Ryzin [12] give a thorough discussion of the model and study the structural properties of the optimal policy. Feng and Gallego [8] consider the problem of deciding the optimal timing of a single price change from a given price to either a given higher or lower second price and show that the optimal policy has a threshold. Feng and Xiao [9] then extend it to a dynamic pricing model with multiple predetermined prices. A dynamic pricing model with multiple prices and reversible price changes is dis-

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cussed in Feng and Xiao [10]. Chatwin [6] also considers the case of multiple prices and extends the results to the case in which the prices and corresponding demand intensities depend on the time-to-go and the case in which the retailer can restock to meet demand at a unit cost after the initial inventory has been sold. Zhao and Zheng [19] consider the model where customers' reservation price distribution changes over time and identify a sufficient condition under which the optimal price decrease over time for a given inventory level. However, the dynamic pricing problem based on discounted criterion is not investigated. Further, we prove that the optimal revenue function is concave in time horizon when the demand process is nonhomogeneous, which is not mentioned under expected criterion.

This paper makes three contributions. The first one is to consider the discounted criterion case and analyze the structural properties of the optimal revenue function and the optimal price. The second one is to provide a method to analyze the structural properties of the optimal revenue function by using the Hamilton–Jacobi–Bellman equation. The third one is to prove the concavity of the optimal revenue function which is different to that proved by Zhao and Zheng [19] who take sample path argument. Our findings include that the optimal revenue function under discounted criterion is increasing in the given time, the number of items left, but decreasing in the discount rate respectively either in finite or infinite horizon. Further, we find out that the optimal price under discounted criterion has a lower bound and is decreasing in the number of items left and the discount rate, but it is increasing over the times.

The rest of the paper is organized as follows. Section 2 formulates the basic finite horizon dynamic pricing model and derives the Hamilton–Jacobi–Bellman equation. Section 3 analyzes the structure properties of the optimal revenue function and the optimal price. Section 4 extends the basic model to a dynamic pricing model with time varying demand and discount rate, and also extends to the infinite horizon model. In Section 5, we conduct numerical examples. Section 6 concludes this paper.

2. Model

Consider a firm which sells N items in a finite time horizon $[T,0]$. Here we use a reversed time index $T>0$. T is the length of the sales horizon for the items, and $t\in[T,0]$ means that a time point when there are t units time to sell the left items. The firm can influence demand by varying its price p and demand follows a non-homogeneous Poisson process. The firm controls the intensity of the non-homogeneous Poisson demand $\lambda_t=\lambda(p_t)$ at time t by using pricing policy p_t. Let N_s be the number of items sold up to time s. A demand is realized at time s if $dN_s=1$, in which case the firm sells one item and receives revenue of p_s. Obviously, the stock N is related with time t and is decreasing in t. Denote $N(t)=n$ that there exist n items to sell for t units time. We denote by $\mathcal{U}(n,t)$ the class of all pricing policies satisfying $\int_0^t dN_s\leqslant n$ which indicates that there is not items to be sold and $p_s\in\mathcal{P}$. Here $\mathcal{P}\subset(0,+\infty)$ is a compact set. The most common examples of a compact set include a discrete set and a closed interval. Here, we only consider the case that $\lambda(p)$ is a continuous, strictly decreasing function of p.

Let $J_\beta(n,t)$ be the supremum of the expected discounted revenue from pricing policy $u\in\mathcal{U}(n,t)$ over $[t,0]$ with stock $N(t)=n$, we have

$$J_\beta(n,t)=\sup_{u\in\mathcal{U}(n,t)}\left\{J_\beta^u(n,t)=E_u\left[\int_0^t e^{-\beta s}p_s dN_s\right]\right\},\tag{1}$$

where β is the discount rate. Obviously, we have $J_\beta(N,0)=0$ and $J_\beta(0,T)=0$.

Gihman and Skorohod [13] showed Eq. (1) has an optimal nonrandomized Markovian policy and the number of items left $N(t)=n$ is the state variable at the time point t (here t is also the state variable). It can be characterized by the pricing decision, which is a function from $\{0,1,\ldots,n\}\times[t,0]$ to $\mathcal{P}$. Next we only consider the nonrandomized Markovian policies.

We consider what happens over a small interval time δt. Suppose the initial state is (n,t), if we select the intensity price $\lambda(p)$ by choosing the price p, then we sell out one item during the δt time interval with probability $\lambda(p)\delta t$, the state becomes $(n-1,t-\delta t)$ and we obtain revenue p for per unit item; with probability $1-\lambda(p)\delta t$, the state becomes $(n,t-\delta t)$ for no selling out item; and with probability $o(\delta t)$, the state becomes others. Hence, we have the following equation by the Principle of Optimality and discounted criterion.

$$\begin{aligned}J_\beta(n,t)=\sup_p\big\{&\lambda(p)\delta t\big[p+e^{-\beta\delta t}J_\beta(n-1,t-\delta t)\big]\\&+[1-\lambda(p)\delta t]e^{-\beta\delta t}J_\beta(n,t-\delta t)+o(\delta t)\big\}.\end{aligned}\tag{2}$$

Rearranging the terms in Eq. (2), we have

$$\begin{aligned}\frac{J_\beta(n,t)e^{\beta\delta t}-J_\beta(n,t-\delta t)}{\delta t}=\sup_p\big\{&\lambda(p)\big[e^{\beta\delta t}p+J_\beta(n-1,t-\delta t)\\&-J_\beta(n,t-\delta t)\big]+o(\delta t)\big\}.\end{aligned}\tag{3}$$

Let $\delta t\to0$, we can derive the Hamilton–Jacobi–Bellman equation as follows

$$\frac{\partial J_\beta(n,t)}{\partial t}+\beta J_\beta(n,t)=\sup_p\big\{\lambda(p)\big[p+J_\beta(n-1,t)-J_\beta(n,t)\big]\big\}.\tag{4}$$

Hence, the firm's optimal policy is to choose a price p which maximizes the right hand side of Eq. (4).

For the notational convenience, let

$$\varDelta J_\beta(n,t)=J_\beta(n,t)-J_\beta(n-1,t),\tag{5}$$

$$b_\beta(p,n,t)=\lambda(p)(p-\varDelta J_\beta(n,t)).\tag{6}$$

The first equation is the marginal revenue of the nth item. Therefore, we have the following proposition.

Proposition 1. *$\varDelta J_\beta(n,t)\leqslant p_{max}$, where p_{max} is the largest price in $\mathcal{P}$.*

Proof. For the state $(n-1,t)$, let u be the optimal policy for selling out n items in the t time horizon, then we have $J_\beta(n,t)=J_\beta^u(n,t)$ by the definition of Eq. (1). Obviously, we have $J_\beta^u(n-1,t)\geqslant J_\beta^u(n,t)-p_{max}$. Therefore, we have $J_\beta(n-1,t)\geqslant J_\beta(n,t)-p_{\max}$, rearranging it, we have $\varDelta J_\beta(n,t)\leqslant p_{\max}$. □

The proposition indicates that the marginal revenue for fixed time horizon t cannot be greater than the largest price in admissible price policy, which is consist with our intuition.

We have assumed that $\lambda(p)$ is a continuous function of p. Thus, $b_\beta(p,n,t)$ in Eq. (6) is also continuous in p. Noting that $\mathcal{P}$ is a compact set, the supremum in (4) can be achieved in $\mathcal{P}$, and we can rewrite HJB equation as

$$\frac{\partial J_\beta(n,t)}{\partial t}+\beta J_\beta(n,t)=\max_{p\in\mathcal{P}}[\lambda(p)(p-\varDelta J_\beta(n,t))].\tag{7}$$

It is obvious that $J_\beta(0,t)=J_\beta(n,0)=0$ for $t\in[T,0]$ and $N(t)=n$ which belongs to $\{0,1,2\ldots,N\}$.

3. Structural properties

In this section, we discuss the structural properties of the optimal revenue function and pricing policy. Firstly, we analyze the properties for the optimal revenue function $J_\beta(n,t)$.

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