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Single vehicle routing with a predefined customer sequence and multiple depot returns

P. Tsirimpas, A. Tatarakis, I. Minis, E.G. Kyriakidis *

Department of Financial and Management Engineering, University of the Aegean, 31 Fostini Street, 82100 Chios, Greece

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Abstract

The optimal routing of a single vehicle with limited capacity that delivers one product to *n* clients according to a predefined client sequence can be determined using dynamic programming. In the present paper we propose and investigate three practical variations of this problem: (i) the case of multiple-product deliveries when each product is stored in its own compartment in the vehicle, (ii) the case of multiple-product deliveries when all products are stored together in the vehicle's single compartment, and (iii) the case in which the vehicle picks up from and delivers a single product to each customer. Suitable dynamic programming algorithms that find the optimal routing of the vehicle are developed for each case. The efficiency of the algorithms is studied by solving large problem sets.

Keywords: Logistics; Routing with pick up and delivery; Routing with multiple-product delivery; Dynamic programming

1. Introduction

Distribution is a key logistics activity and is responsible, in general, for a significant part of total logistics costs (see [1]) for a manufacturing or logistics-focused company. In some cases, such as in the soft drink industry, distribution costs represent approximately 70% of the value added costs of goods (see [8]). The potential for savings by optimizing delivery, i.e. finding the optimal set of vehicle schedules and routes, is therefore considerable, since the cost of fleet operations depends heavily on the efficiency of the routes of the delivery vehicles.

A large body of literature has focused on vehicle routing optimisation, and a wide spectrum of theoretical and practical routing problems have been considered. Two fundamental problems in this area are the travelling salesman problem (TSP) and the vehicle routing problem (VRP). The former, possibly the most widely investigated problem in combinatorial optimization, involves finding an optimal route for visiting *n* cities and returning to the point of origin; in this case the inter-city distances are symmetric and satisfy the triangular

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* Corresponding author. Tel.: +30 2271035464.
E-mail addresses: fme01028@fme.aegean.gr (P. Tsirimpas), a.tatarakis@fme.aegean.gr (A. Tatarakis), i.minis@fme.aegean.gr (I. Minis), kyriak@fme.aegean.gr, kyriak@aegean.gr (E.G. Kyriakidis).

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inequality. Many algorithms have been developed to address this *NP*-hard problem. Lawler et al. [13] provide a comprehensive introduction to the TSP, and Bentley [2], as well as Golden and Stewart [7], have presented in depth surveys of the available solution algorithms (see also Part I in [4]).

The VRP refers to a fleet of vehicles that are based at one or several depots, and deliver goods to a set of n customers comprising the nodes of a predefined network. Each vehicle starts at the depot, delivers goods to a subset of these customers, fully satisfying their demand and, finally, returns to the depot. The route of each vehicle satisfies a number of constraints. For instance, the load carried by each vehicle should not exceed the vehicle's capacity; network related constraints are also necessary, such as serving each customer exactly once, not allowing subtours, etc. The objective is to minimize the overall distance travelled. VRP is also an NP-hard problem and several exact algorithms, heuristics and metaheuristics have been proposed to provide efficient solutions (see [4–6,10,12,20]).

In this paper we address a special case of delivery: We consider a single vehicle serving customers at a predefined sequence; the vehicle is allowed to return to the depot for stock replenishment. Thus, the only decision to be made is at which points along the route the vehicle will return to the depot for stock replenishment. We refer to this problem as the vehicle routing with depot returns (for stock replenishment) problem (VRDRP). In this problem the demand of each customer, as well as the distances among all network nodes are known in advance. The quantity that can be loaded to the vehicle cannot exceed the capacity of the vehicle.

It is clear from the above discussion that upon completion of service at each customer point (site), the vehicle has to either: (a) travel to the next customer, as long as the demand of the next customer is not greater than the remaining stock on board, or (b) return to the depot in order to refill, and resume the route (see Fig. 1). Note that even if there is adequate stock aboard the vehicle to serve the next customer, it may be advantageous to drive to the depot to refill, if this minimizes the distance of the entire route; an obvious such case is if the site is very close to the depot, and the vehicle has significant empty capacity.

A route in the network may be modeled by a sequence of 0's and 1's. The value '0' at position i of this sequence represents the decision to travel to customer $i+1$ directly after serving customer i, without visiting the depot; the value '1' represents the decision to visit the depot after serving customer i, in order to replenish its stock, and then visit customer $i+1$. As an example, the sequence $[0,0,0,1,0,0,0,1]$ represents a route in a network that includes eight customers. After serving customers 4 and 8 the vehicle returns to the depot for stock replenishment. If the number of customers in the network is equal to n, then there are 2^{n-1} possible '0–1' combinations, and, therefore, 2^{n-1} feasible routes, since the last digit in the sequence is always 1. Out of these, only a subset is feasible if the demand-capacity restrictions are applied.

Practical applications of this problem may arise in different settings. Material handling systems in a manufacturing shop often operate along fixed pathways that connect the material warehouse with workcenters located along this pathway. For example, automated guided vehicle systems (AGVs) are self propelled vehicles typically guided along a magnetic induction strip, or a painted strip on the shop floor, and transport discrete parts to workcenters, obviously in a predefined sequence. Note that in addition to the main pathway connecting the workcenters, there are spurs connecting each workcenter with the material warehouse, allowing the

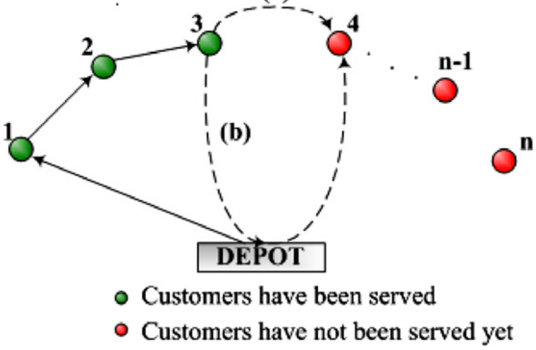

Fig. 1. Decision at each customer point.