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Existence of equilibria in a decentralized two-level supply chain

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ABSTRACT

In this paper, we analyze equilibria in competitive environments under constraints across players' strategies. This means that the action taken by one player limits the possible choices of the other players. In this context, the usual approach to show existence of equilibrium, Kakutani's fixed point theorem, cannot be applied directly. In particular, best replies against a given strategy profile may not be feasible. We devise a new fixed point correspondence to deal with the feasibility issue.

Our main motivation to study this problem of co-dependency comes from the field of supply chain planning. A set of buyers is faced with external demand over a planning horizon, and to satisfy this demand they request inputs from a set of suppliers. Both suppliers and buyers face production capacities and they plan their own production in a decentralized manner. A well-known coordination scheme for this setting is the upstream approach where the plan of the buyers is used to decide the request to the suppliers. We show the existence of equilibria for a (shared) inventory cost minimization version of this coordination scheme in which a distribution center manages the inventory of the inputs. However, we illustrate with an example that the centralized solution is not, in general, an equilibrium, suggesting that regulation may be needed.

1. Introduction

In a non-cooperative game we are given a set of players and their corresponding strategy sets. Players simultaneously choose their strategies, and the payoff of each player depends on the strategies chosen. Because of this structure of the payoffs, most of the literature devoted to non-cooperative games assumes that the feasible strategies of each player are independent of the strategies chosen by the other players. However, there are many relevant situations where constraints exist across the strategies of the players.

As a first leading (simple) example, one may think of the following congestion game. Suppose there are ten tourists visiting city *C*. City *C* has two hotels, hotel *X* and hotel *Y*. Both hotels have room for seven guests, so both hotels together can accommodate all tourists. The preference of each tourist is to stay in a hotel with as few other tourists as possible. Clearly, in each equilibrium five tourists stay in hotel *X* and five tourists stay in hotel *Y*.

However, rooms have to be booked in advance. And both hotels have several ways to make a reservation, such as by phone, by fax, or via internet. Each tourist has two strategies (namely make a reservation in hotel *X*, or make a reservation in hotel *Y*), but the feasibility of executing such a strategy does not depend on this strategy alone, but also on the actions taken by the other tourists. If everyone else books in hotel *Y*, booking in hotel *X* is a very good strategy. However, as soon

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as seven other tourists decide to book in hotel X, doing this as well results in an infeasible allocation. And a tourist cannot tell the difference in advance.

In this paper, we consider an infeasible allocation as an undesirable outcome for all parties concerned. There are many ways to explicitly model undesirability of infeasible allocation. We can, for example, add extra stages to the basic game. In the hotel example one could build in an extra renegotiation stage in which the hotels communicate with each other and redirect tourists. This implies that extra time and effort is wasted (the hotel management has to make extra phone calls and the tourists have to carry their suitcases over to the other hotel), which will be avoided in equilibrium. Another way to redefine the basic game is to incorporate the extra costs directly in the utility function of the tourists.

However, no matter how we model infeasibilities, the aim is always to augment the game in such a way that the, let us say, natural equilibria indeed become equilibria, and that infeasible allocations are not chosen in equilibrium. In the congestion game, whether we add extra stages to the game or penalize infeasibilities, the aim is to augment the game in such a way that each hotel accommodates five tourists in each equilibrium of the augmented game. We could achieve that by, for example, saying that for a feasible allocation the utility of a tourist equals 7 minus the number of tourists that book the same hotel, while the utility of having booked an overbooked hotel equals -1.

Provided that we augment the game appropriately, we do not need to specify how infeasibility can, might, or ought to be resolved. On the contrary our main point here is that, even though there may be many ways to resolve infeasibilities (adding more stages to the game, or use monetary punishments for infeasible choices), *in equilibrium* players avoid infeasibilities. It is in that sense that the specific way in which infeasibilities are treated is not of interest to us. When the players manage to orchestrate their actions and play an equilibrium, infeasibility simply does not happen as we can clearly see in the congestion game.

Also on a more theoretical level feasibility restrictions across player's strategies have been studied before. For example, Caron and Laye (2003) consider a Cournot oligopoly model where market demand puts constraints on the total quantity produced by all firms. In the context of congestion games, Correa et al. (2004) study efficiency of Wardrop equilibria in capacitated networks, where the capacity constraints impose feasibility restrictions across flows between origin–destination pairs in the network. In the context of strategic voting, Saporiti and Tohmé (2006) consider a voting model where the assumption of single crossing imposes restrictions across individual preferences.

Our initial motivation to study constrained competitive environments comes from decentralized decision making in supply chain planning, see e.g. Cachon and Netessine (2004) for non-cooperative games in supply chain analysis. We consider a two-level supply chain composed by suppliers at the first level and buyers at the second level. The buyers are faced with deterministic and dynamic external demand over a planning horizon, and to satisfy this demand they request inputs from the suppliers. Both suppliers and buyers face linear production and inventory holding costs as well as production capacities. This supply chain functions in a decentralized manner where information such as unit costs and capacities are only locally known. It is evident that when the suppliers plan their production, they will, in general, be constrained by the plan of the buyers. Indeed, the demand suppliers need to satisfy, and its timing, depends on the amount they get granted from the buyers' requests. Similarly, the plan of the buyers is constrained by the production decisions taken by the suppliers.

Cachon and Netessine argue that the game theoretic analysis of such models is problematic. In this paper, we show that nevertheless equilibria in competitive environments under constraints across players' strategies do exist. However, Kakutani's fixed point theorem cannot be applied directly in this context. In particular, best replies against a given strategy profile may not be feasible. We devise a new fixed point correspondence to deal with the feasibility issue. Under the usual conditions, upper semicontinuity and convex-valuedness of best response correspondences, we show the existence of Nash equilibrium.

In the context of decentralized decision making in a supply chain, the upstream planning is one of the most well-known coordination schemes, see Bhatnagar et al. (1993). In the one-supplier one-buyer setting, the buyer plans his production and passes his plan upstream the supply chain, i.e., to the supplier. The supplier has to plan his production according to the requirements defined by the plan of the buyer. Dudek and Stadtler (2005) aim to improve this mechanism and propose a coordination scheme based on negotiations having as a starting point the upstream solution. In our competitive supply chain setting, where there will be several suppliers for the same product, a distribution center manages the inventory of the inputs and the corresponding costs are shared.

The distribution center receives requests from the buyers and the only information offered by the suppliers are upper bounds on production levels in each period, and its goal is to coordinate the supply chain by means of a upstream planning. However, in the presence of multiple suppliers, the upstream mechanism is a whole class depending on how the requirements of the buyers are allocated to the suppliers. We propose a plausible allocation based on fulfilling the requirements of the buyers as late as possible, thus minimizing the (shared) inventory costs. We show that, for this upstream planning mechanism, a Nash equilibrium exists. However, we illustrate with an example that the centralized solution is not, in general, an equilibrium, suggesting that regulation may be needed.

This paper is organized as follows: in Section 2, we introduce a game played in a constrained environment. Under the assumption of convex-valuedness and upper semicontinuity of the best response correspondence, we show the existence of a Nash equilibrium. In Section 3, we present the decentralized supply chain setting that motivated this work. In Section 4, we propose an upstream coordination mechanism based on (shared) inventory costs minimization, and show that for this constrained environment Nash equilibrium exists. In Section 5, we illustrate with examples that the centralized planning is not, in general, a Nash equilibrium and that the equilibrium is not unique. We derive an upper bound on the cost difference