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European Journal of Operational Research 195 (2009) 358–363

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

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Production, Manufacturing and Logistics

Optimal manufacturer's replenishment policies in the EPQ model under two levels of trade credit policy

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Received 18 January 2007; accepted 2 February 2008
Available online 9 February 2008

Abstract

In 2007, Huang proposed the optimal retailer's replenishment decisions in the EPQ model under two levels of trade credit policy, in which the supplier offers the retailer a permissible delay period M, and the retailer in turn provides its customer a permissible delay period N (with $N < M$). In this paper, we extend his EPQ model to complement the shortcoming of his model. In addition, we relax the dispensable assumptions of $N < M$ and others. We then establish an appropriate EPQ model to the problem, and develop the proper theoretical results to obtain the optimal solution. Finally, a numerical example is used to illustrate the proposed model and its optimal solution.

Keywords: Inventory; EPQ; Trade Credits; Permissible delay in payments

1. Introduction

Goyal (1985) first developed an economic order quantity (EOQ) model under the conditions of permissible delay in payments. Aggarwal and Jaggi (1995) extended Goyal's model to the case of deterioration. Jamal et al. (1997) analyzed Aggarwal and Jaggi's model to allow for shortages. Teng (2002) amended Goyal's model by considering the difference between unit price and unit cost and established an easy analytical closed-form solution to the problem. Chung and Huang (2003) proposed an economic production quantity (EPQ) inventory model for a retailer when the supplier offers a permissible delay in payments by assuming that the selling price is the same as the purchase cost. Huang (2003) extended Goyal's model to develop an EOQ model in which supplier offers the retailer the permissible delay period M (*i.e.*, the supplier trade credit), and the retailer in turn provides the trade credit period N (with $N \leqslant M$) to its customers (*i.e.*, the retailer trade credit). Huang (2007) incorporated both Chung and Huang (2003) and Huang (2003) to investigate the optimal retailer's replenishment decisions with two levels of trade credit policy in the EPQ framework. Jaggi et al. (2008) incorporated the concept of credit-linked demand and developed an inventory model under two levels of trade credit policy to determine the optimal credit as well as replenishment policy jointly for the retailer. Ho et al. (2008) formulated an integrated supplier–buyer inventory model with the assumption that the market demand is sensitive to the retail price and the supplier adopts a trade credit policy to determine the optimal pricing, shipment and payment policy. Lately, Chang et al. (in press) reviewed the contributions on the literature in modeling of inventory lot-sizing under trade credits.

Huang (2007) proposed the optimal retailers replenishment decisions in the EPQ model under two levels of trade credit policy. He then developed the theoretical results. However, he ignored the fact that the retailer (or manufacturer) offers its customers a permissible delay period N, hence, the retailer receives its revenue from N to $T+N$, not from 0 to T as shown in Huang's model formulations.

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doi:10.1016/j.ejor.2008.02.001

In this paper, we not only extend his EPQ model to complement the above mentioned shortcoming but also relax some dispensable assumptions of $N < M$ and others. In our view the permissible delay period offered by the retailer (N) is independent of the permissible delay period offered by the supplier (M) to the retailer. The retailer based on the prevalent market conditions must choose the appropriate value of N. In many situations retailers are forced to offer a permissible delay period to their customers while receiving no permissible delay period ($M = 0$) from their suppliers.

We then propose the generalized formulation to the problem, and establish the theoretical results to obtain the optimal solution. Finally, a numerical example is given to illustrate the proposed model and its optimal solution.

2. Mathematical formulation

The following notation will be adopted are the similar as those in Huang's EPQ model under trade credit.

D	the demand rate per year
P	the replenishment rate (*i.e.*, production rate) per year, $P \geqslant D$
A	the ordering (or set-up) cost per order (lot)
ρ	$1 - \frac{D}{P} \geqslant 0$, the fraction of no production
c	the unit purchasing price
s	the unit selling price, $s \geqslant c$
h	the unit stock holding cost per item per year excluding interest charges
I_e	the interest earned per dollar per year
I_k	the interest charged per dollar in stocks per year by the supplier
M	the manufacturer's trade credit period offered by supplier in years
N	the customer's trade credit period offered by manufacturer in years
T	the cycle time in years
TVC(T)	the annual total relevant cost, which is a function of T
T^*	the optimal cycle time of TVC(T)
Q^*	the optimal lot size of TVC(T)

Huang (2007) assumed that $I_k \geqslant I_e$ and $M \geqslant N$. In this note, we will relax these two dispensable assumptions. The other assumptions are the same as those in Huang (2007). The annual total relevant cost consists of the following elements:

1. Annual ordering cost $= \frac{A}{T}$
2. Annual stock holding cost $= \frac{hDT\rho}{2}$
3. According to assumptions (6) and (7) in Huang (2007), as well as the values of N and M, there are two cases (Case 1: $N \leqslant M$ and Case 2: $N \geqslant M$) to occur in interest charged and interest earned per year.

Case 1. $N \leqslant M$

The manufacturer buys all parts at time zero and must pay the purchasing cost at time M. Based on the values of M (*i.e.*, the time at which the manufacturer must pay the supplier to avoid interest charge) and $T + N$ (*i.e.*, the time at which the manufacturer receives the payment from the last customer), we have two possible sub-cases. Sub-case 1-1: $T + N \geqslant M$ and Sub-case 1-2: $T + N < M$. Now, let us discuss the detailed formulation in each sub-case.

Sub-case 1-1. $M \leqslant T + N$

In this sub-case, the manufacturer pays off all units sold by $M - N$ at time M, keeps the profits, and starts paying for the interest charges on the items sold after $M - N$. The graphical representation of this sub-case is shown in Fig. 1. However, the manufacturer can not payoff the supplier by M because the supplier credit period M is shorter than the customer last payment time $T + N$. Hence, the manufacturer must finance all items sold after time $M - N$ at an interest charged I_k per dollar per year. The interest charged per cycle is cI_k times the area of the triangle BCD shown in Fig. 1. Therefore, the interest charged per year is given by

$$\frac{cI_k}{T}\left\{\frac{D[T+N-M]^2}{2}\right\}. \tag{1}$$

Notice that Huang (2007) did not recognize that the last customer buys the product at time T, and pays the manufacturer at time $T + N$ due to its customer trade credit period N. Consequently, he obtained the interest charged per year as

$$\frac{cI_k}{T}\left[\frac{\rho(DT^2 - PM^2)}{2}\right] \text{ or } \frac{cI_k}{T}\left[\frac{D(T-M)^2}{2}\right],$$

which is different from ours in (1).

On the other hand, the manufacturer starts selling products at time 0, but getting the money at time N. Consequently, the manufacturer accumulates revenue in an account that earns I_e per dollar per year starting from N

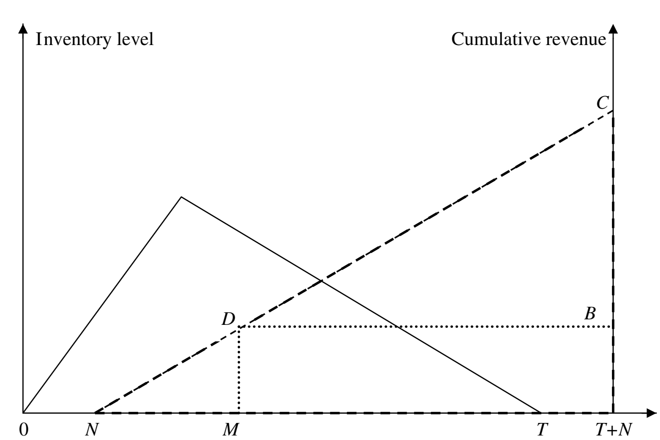

Fig. 1. $M \leqslant T + N$.