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A game theory approach in seller–buyer supply chain

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ABSTRACT

In this paper, several seller–buyer supply chain models are proposed which incorporate both cost factors as well as elements of competition and cooperation between seller and buyer. We assume that unit marketing expenditure and unit price charged by the buyer influence the demand of the product being sold. The relationships between seller and buyer will be modeled by non-cooperative and cooperative games, respectively. The non-cooperative game is based on the *Stackelberg strategy solution concept*, where we consider separately the case when the seller is the leader (*Seller-Stackelberg*) and also when the buyer is the leader (*Buyer-Stackelberg*). *Pareto efficient* solutions will be provided for the cooperative game model. Numerical examples presented in this paper, including sensitivity analysis of some key parameters, will compare the results between different models considered.

1. Introduction

A seller–buyer supply chain represents a manufacturer which wholesales a product to a retailer, who, in turn retails it to a consumer (Yang and Zhou, 2006; Chen et al., 2006; Dai et al., 2005). In the literature, the terms vendor, supplier, and manufacturer have been used interchangeably to represent the seller. Likewise, the word retailer has been used to represent the buyer. In this paper, for the sake of simplicity, we will use the nomenclature buyer and seller. The related literature on finding optimal seller and buyer's policy of production and ordering can be broadly categorized into three groups based on the following assumptions: seller–buyer interaction is seen in light of constant demand, seller and buyer are independently studied where demand varies, and seller and buyer interaction is considered without logistic cost including setup/ordering and holding/carrying costs. We briefly summarize these models in order to compare with our proposed approach.

There are many possible interactive coordination mechanisms that can occur between the two members of a seller–buyer supply chain. Various types of mechanisms have been discussed in the literature on supply chain coordination such as quantity discount, credit option, buy back or return policies, quantity flexibility and commitment of purchase quantity (Sarmah et al., 2006). Quantity discount, a popular tool of coordination mechanism, is considered in Chiang et al. (1994), Corbett and de Groote (2000) and Viswanathan and Wang (2003). Abad (1994) proposed a model of seller–buyer relationship, where demand is price sensitive and provided procedure of finding the optimal policy for both seller and buyer under a cooperative scenario. A similar model was presented in Abad and Jaggi (2003) where the main assumption is that the seller offers trade credit to the buyer. Several works (Sucky, 2005, 2006; Chan and Kingsman, 2007; Heuvel et al., 2007; Dai and Qi, 2007) have addressed the problem of determining the optimal order quantity (lot size) or order (production) cycles in a cooperative structure in order to achieve maximum savings or enhance profit for the whole supply chain where demand rate is considered fixed.

In contrast, fixed demand is avoided in some research where joint lot sizing and pricing decisions are used to determine the optimal price and order quantity for maximization of the firm's profit. In such cases, price would depend on demand over a planning horizon (Abad, 1994; Lee, 1993; Lee et al., 1996; Kim and Lee, 1998; Jung and Klein, 2001, 2005). Similar approaches have also been used in cases where both marketing expenditure and price influence demand (Freeland, 1982; Lee and Kim, 1993, 1998; Sajadi et al., 2005). A significant shortcoming of all these models is that they only regard seller or buyer supply chain management problem without considering any interaction between buyer and seller.

The effort expended in marketing product is also an important coordination mechanism tool that can occur between the two members of a seller–buyer supply chain. For example, Kotler (1997) noted that effort expanded in marketing is an expenditure which can be spent in several ways such as advertising, sales

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promotion, sales force and marketing research expenditure. Huang and Li (2001) and Li et al. (2002) investigated manufacturer–retailer coordination as a cooperative advertising in supply chain problem. They highlighted the impact of investment in brand name, local advertising and sharing policy in three models under a cooperative regime in which the seller agrees to share a fraction of the total local advertising expenditure with the buyer. Yue et al. (2006) proposed a similar model under the assumption that seller offers a price reduction to customers. The profit function in a typical supply chain model contains a logistic cost component. However, in order to avoid the confounding effect of logistic cost, these papers assume that lot size is equal to demand.

In this paper, we propose several models of seller–buyer relationship subject to the impact of marketing effort. In our models, the seller produces a product and wholesales it to the buyer, who then retails the product to the consumer. The production rate of the seller is assumed to be linearly related to the market demand rate, while demand is sensitive to selling price and marketing expenditure. When demand is sensitive to selling price, the unit price imposed by the seller on the buyer does influence the end demand for the product. In such case, if the buyer expends on marketing, then the seller needs to coordinate her pricing and lot sizing to cope with new demand for the product. In this paper, we consider such interaction among the seller and buyer using non-cooperative as well as cooperative game theory. The non-cooperative game aspect will be considered from two perspectives: *Seller-Stackelberg* and *Buyer-Stackelberg* scenarios. In the first scenario, the seller dominates the buyer in a conventional way, whereas in the second, power has shifted from the seller to the buyer. Similarly, we consider the cooperative game model and Pareto efficient solutions will be provided for this model. We will demonstrate that a buyer or seller could benefit more from a cooperative structure than a non-cooperative one.

Another novel aspect of our models is to allow the sellers to impose lot sizes on the buyers. Conventionally, buyer determines lot size but in many large industries such as heavy equipment industry, aerospace industry, automotive industry where highly specialized equipments are supplied to customers, the cost of production is high and in order to reduce this and optimize production, it is common for the seller to fix lot sizes. For example, Kelle et al. (2003) investigate two scenarios for shipment quantity, fixed by the supplier or buyer in a Just in Time (JIT) supply chain system. A buyer in a JIT system would like small, frequent shipments whereas a seller prefer large production lot sizes. The paper compares and contrasts the costs when either the supplier or the seller holds a dominant position and is able to impose lot sizes on the others.

The remainder of this paper is organized as follows. We give the notation and assumptions underlying our models in Section 2. This section also formulate the problem, including a discussion of the model from the buyer's and seller's perspective. In Section 3, the non-cooperative Seller-Stackelberg and Buyer-Stackelberg models are discussed. The cooperative game aspect is presented in Section 4. Section 5 presents some computational results including a number of numerical examples and their sensitivity analysis. Finally, the paper concludes in Section 6 with some suggestions for future work in this area.

2. Notation and problem formulation

This section introduces the notation and formulation used in our supply chain problem. Specifically, all decision variables, input parameters and assumptions underlying our models will be stated.

2.1. Decision variables

$\mathscr{V}$	the price charged by the seller to the buyer (\$/unit)
Q	lot size (units) determined by the seller
P	selling price charged by the buyer (\$/unit)
M	marketing expenditure incurred by the buyer (\$/unit)

2.2. Input parameters

k	scaling constant for demand function $(k > 0)$
u	scaling constant for production function $(u \geqslant 1)$
i	percent inventory holding cost per unit per year
α	price elasticity of demand function $(\alpha > 1)$
β	marketing expenditure elasticity of demand $(0 < \beta < 1, \beta + 1 < \alpha)$
A_b	buyer's ordering cost (\$/order)
A_S	seller's setup (ordering cost) (\$/setup)
C_S	seller's production cost including purchasing cost (\$/unit)
r	seller's production rate (units/day)
$D(P, M)$	annual demand; for notational simplicity we let $D \equiv D(P, M)$
d	market demand rate (units/day)

Since we have assumed that the seller's setup cost is high relative to the buyer's, we let $A_S > A_b$. Also, our demand is assumed to be a function of P and M and is based on Lee and Kim (1993):

$$D(P, M) = kP^{-\alpha}M^{\beta}. \tag{1}$$

The fact that our demand function is not constant, as is assumed in standard deterministic inventory models, is fairly common in many current models, see, e.g. Abad (1994), Lee (1993), Lee et al. (1996), Kim and Lee (1998), Jung and Klein (2001, 2005).

2.3. Assumptions

The proposed models in this paper are based on the following assumptions:

1. Planning horizon is infinite.
2. Parameters are deterministic and known in advance.
3. Even though the buyer usually determines lot size in conventional supply chain models, here we assume in the contract between seller–buyer that the seller determines lot size. This is appropriate in circumstances where setup, inventory and internal storage costs are high for the seller relative to the buyer. Therefore, our model is restricted to cases where there is a low number of providers and few competitors.
4. The annual demand depends on the selling price and marketing expenditure according to (1).
5. Shortages are not permitted, hence the production rate r is greater than or equal to demand rate d and, without loss of generality, we will assume them to be linearly related by the following equation:

$$r = ud, \quad u \geqslant 1. \tag{2}$$

2.4. The Buyer's model formulation

The buyer's objective is to determine the selling price and marketing expenditure such that her net profit is maximized. The selling price and marketing expenditure influence the demand and consequently, the seller's lot size. Here, we consider the buyer's model proposed by Abad (1988) but with the addition of the marketing expenditure, M as an additional decision variable. Therefore, the buyer's annual profit function is