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Discrete Optimization

Minimizing due date related measures for a single machine scheduling problem with outsourcing allowed

Ik Sun Lee, C.S. Sung *

Department of Industrial Engineering, KAIST, Yuseong-gu, Daejeon 305-701, Republic of Korea

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Abstract

This paper considers two single-machine scheduling problems with outsourcing allowed where each job can be either processed on an in-house single-machine or outsourced. They include the problem of minimizing maximum lateness and outsourcing costs, and that of minimizing total tardiness and outsourcing costs. Outsourcing is commonly required as a way to improve productivity in various companies including electronics industries and motor industries. The objective is to minimize the weighted sum of the outsourcing cost and the scheduling measure represented by either one of maximum lateness and total tardiness, subject to outsourcing budget. It is proved that the problem is NP-hard. Some solution properties are characterized to derive heuristic algorithms, and also a branch-and-bound algorithm. Numerical experiments are conducted to evaluate performance of the derived algorithms.

Keywords: Single-machine scheduling; Due-date related measures; Outsourcing strategy

1. Introduction and problem description

Outsourcing is commonly required as a way to improve production scheduling effectively and efficiently in various companies including electronics industries and motor industries. This provides the authors with the motivation of considering the proposed scheduling problem with outsourcing allowed. The objective of the problem is to minimize the weighted sum of the outsourcing cost and the scheduling measure, subject to outsourcing budget, where the scheduling measure is represented by either one of maximum lateness and total tardiness.

This paper considers an outsourcing situation where outsourcing is allowed in parallel to the associated in-house scheduling so as to promote the overall scheduling quality in terms of due date satisfaction. For example, in a situation where demands (scheduling requests) requested from customers are too much to be taken care of by a given manufacturing (scheduling) capacity and the associated due dates are also too tight,

* Corresponding author. Tel.: +82 42 869 3131; fax: +82 42 869 3110.
E-mail addresses: lis1007@kaist.ac.kr (I.S. Lee), cssung@kaist.ac.kr (C.S. Sung).

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outsourcing assignment may be positively considered in the way of allocating the associated portion of work substitute to another in-house manufacturing division or some other companies. In such a situation, outsourcing may incur certain charges associated with outsourcing ordering and lead-time. However, such outsourcing charges may be compensated for by fulfilling customer demands (due date satisfaction) on time so as to make more profit.

In the literature, a few researches have been done in association with outsourcing strategy. For example, Cachon and Harker (2002) and Kaipia and Tanskanen (2003) have investigated reasons why various firms have adopted outsourcing in their management. Kim (2003) has considered selection of outsourcing resources (companies). Kolisch (2000) has considered a make-to-order production problem with outsourcing strategy allowed, for which he has derived a mixed-integer programming model to minimize holding and setup costs of the associated entire supply chain. However, only few papers have considered scheduling problems with outsourcing allowed. For example, Lee et al. (2002) have considered an advanced planning and scheduling (APS) problem with outsourcing allowed, for which they have derived a genetic algorithm to minimize the makespan objective measure.

The proposed problem is now stated in detail as follows. There are n jobs available at time zero. Let p_j, d_j and o_j denote processing time, due date and outsourcing cost of job j, respectively. Each job can be either processed on an in-house single-machine without preemption allowed or outsourced at cost o_j. O_π is defined as an *outsourcing job set* which is composed of all outsourcing jobs in schedule π. Then, the total outsourcing cost $\mathrm{OC}(\pi)$ associated with schedule π is $\sum_{j\in O_\pi} o_j$. The problem also considers the outsourcing budget constraint $\mathrm{OC}(\pi) \leqslant K$. Moreover, outsourcing lead-time l_j is required for each outsourcing job j, so that the corresponding completion time of the outsourcing job is at time point l_j, such as $C_j = l_j$ for $\forall j \in O_\pi$, where C_j denotes the completion time of job j. Without loss of generality, it is assumed that K, d_j, p_j, o_j and l_j have integer values.

The standard classification scheme for scheduling problems (Graham et al., 1979) $\alpha|\beta|\gamma$ is adopted in this paper where α indicates the scheduling environment, β describes the job characteristics or restrictive requirements, and γ defines the objective function to be minimized. The proposed problem is a single-machine problem, so that $\alpha = 1$ holds. For β, "*Budget*" represents the outsourcing budget constraint "$\mathrm{OC}(\pi) \leqslant K$". For γ, the objective cost function of the proposed problem is the weighted sum of the outsourcing cost and the associated scheduling cost. The outsourcing cost, OC, is expressed as $\sum_{j\in O_\pi} o_j$, and the scheduling cost is represented by one of the followings:

$$L_{\max} = \max_{1\leqslant j\leqslant n}\{C_j - d_j\} \quad \text{(maximum lateness)},$$

$$\sum T_j = \sum \max\{0, C_j - d_j\} \quad \text{(total tardiness)}.$$

Accordingly, the proposed problem is expressed as either one of $1|Budget|(1-\delta)L_{\max} + \delta\cdot\mathrm{OC}$ and $1|Budget|(1-\delta)\sum T_j + \delta\cdot\mathrm{OC}$, where δ denotes a constant cost parameter and $0 \leqslant \delta \leqslant 1$.

2. Minimizing maximum lateness and outsourcing costs

2.1. Problem complexity analysis

This section proves that the problem $1|Budget|(1-\delta)L_{\max} + \delta\cdot\mathrm{OC}$ is NP-hard, as in Theorem 1.

Theorem 1. *The problem* $1|Budget|(1-\delta)L_{\max} + \delta\cdot OC$ *is NP-hard even if* $\delta = 0$.

Proof. The proof can be done in polynomial reduction from the Partition Problem (Garey and Johnson, 1979), which is known to be NP-hard. The Partition Problem is stated as follows:

Given a set $\{a_1, \ldots, a_q\}$ of positive integers, where $\sum_{i=1}^{q} a_i = 2B$ and $0 < a_i < B$, for $i = 1, \ldots, q$, does there exist a subset $S \subset \{1, \ldots, q\}$ such that $\sum_{i\in S} a_i = B$?

Now, consider the following instance of the given problem $1|Budget|(1-\delta)L_{\max} + \delta\cdot\mathrm{OC}$;

$$n = q, \quad K = B, \quad \delta = 0,$$

$$p_j = a_j, \quad o_j = a_j, \quad d_j = l_j = B, \quad j = 1, \ldots, q.$$