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Continuous Optimization

The minimum equitable radius location problem with continuous demand

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Abstract

We analyze the location of p facilities satisfying continuous area demand. Three objectives are considered: (i) the p-center objective (to minimize the maximum distance between all points in the area and their closest facility), (ii) equalizing the load service by the facilities, and (iii) the minimum equitable radius – minimizing the maximum radius from each point to its closest facility subject to the constraint that each facility services the same load. The paper offers three contributions: (i) a new problem – the minimum equitable radius is presented and solved by an efficient algorithm, (ii) an improved and efficient algorithm is developed for the solution of the p-center problem, and (iii) an improved algorithm for the equitable load problem is developed. Extensive computational experiments demonstrated the superiority of the new solution algorithms.

Keywords: Location; p-Center; Equitable loads; Voronoi diagrams

1. Introduction

Recently, problems of locating p facilities to satisfy continuous demand in an area were investigated. Ohya et al. (1984) formulated and solved the p-median problem in an area by applying an iterative procedure on the Voronoi diagram of the facilities (Okabe et al., 2000; Suzuki and Okabe, 1995). A similar approach was utilized by Okabe and Suzuki (1987) to formulate and solve the competitive location problem in an area and by Suzuki and Drezner (1996) to formulate and solve the p-center problem in an area. Baron et al. (2007) suggested an equitable load model (EL) for uniform continuous demand in an area. Their model seeks the location of p facilities in the area, each point in the area is serviced by the closest facility as long as it is within a given coverage distance r from the facility. The objective is equalizing the loads serviced by each facility which is formulated as minimizing the maximum load serviced by the facilities. If the value of r is too small, there may be no feasible solution. For a large enough value of r, the optimal solution is equal in areas for all facilities. They solved the EL problem by an iterative procedure based on Voronoi diagrams.

The question of finding the minimum possible r among all optimal solutions to the equitable load problem, is a practical and interesting problem. We term this problem ‘‘the minimum equitable radius” (MER) problem. The EL solution is usually not unique. Therefore, the maximum distance to the nearest facility may vary from one optimal solution to another. The MER problem is to find the minimum value among those maximum distances.

To illustrate the MER problem consider the location of four facilities in a square. If the four facilities are placed equally spaced on the periphery of a circle centered at the center of the square (the radius of the circle is irrelevant), they divide the

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square into four equal areas (see Fig. 1). This is because the perpendicular bisector between two facilities must pass through the center of the square, and the four areas are thus congruent. Each angle θ and any radius lead to the locations of four facilities which are optimal solutions to the EL problem. Therefore, the number of EL solutions is infinite. It is easy to show that the radius enclosing each shape is $\frac{\sqrt{2}}{2\cos\left(\frac{\pi}{4}-\theta\right)}$. This radius is obtained when the facilities are located at the centers of the lines connecting the intersection points of the perpendicular bisectors with the sides of the square. The optimal solution of the MER problem (the minimum possible radius), as proven in this paper, is obtained for $\theta = \frac{\pi}{4}$ when the facilities are located on the diagonals at the centers of the lines connecting the center of the square and its vertices. The square is divided into four equal small squares.

The p-center problem is investigated in other environments as well. In the plane, it is investigated by Drezner (1984) and Vijay (1985) for Euclidean distances and by Drezner (1987) for rectilinear distances. In a network environment the p-center problem is extensively investigated (for example, Handler, 1990), see, Daskin (1995) and Current et al. (2002) for a review.

The EL problem with no coverage radius constraints was investigated by Drezner and Drezner (2006) for planar problems and by Berman et al. (in press) in the network environment. We are not aware of any paper suggesting the MER problem.

There are many situations in which equal loads of the facilities is desirable. When cell phone towers are established in an area, the busiest tower determines the efficiency of the system thus equalizing loads are desirable. This applies to any design of p identical M/M/1 facilities where minimizing the maximum waiting time at the facilities is equivalent to minimizing the maximum arrival rate at the facilities which is proportional to the number of customers getting service at the facility. When carving market territories to be assigned to marketing representatives, the objective is to assign to each territory an equitable market potential. Another example is the problem of designing machines with similar capacities in a production facility. The throughput of the system depends on the machine with the maximum load. Voter districting is another application. The objective is that each district will have about the same number of voters.

The maximum distance to the closest facility is a measure of good service used in numerous p-center applications. The MER model combines the two objectives. We seek the minimum possible radius among all optimal solutions to the EL model thus achieving both the EL and the p-center objectives. In every application described above for the EL model, it is desirable to minimize the maximum distance to the closest facility in addition to equalizing the loads as much as possible. For example, it is desirable that voters in a district are close to the voting place. It is advantageous for customers to be close to the facilities.

In this paper, we propose three contributions. First, we propose a better heuristic algorithm to solve the p-center problem in an area. Second, we propose a better heuristic algorithm for the solution of the EL problem for a large enough value of r (for example $r \geqslant \sqrt{2}$ for a square area as suggested by Baron et al., 2007) so that the distance constraints are not active in the solution. Third, we propose and heuristically solve the MER problem of finding the smallest possible radius among all equitable loads solutions.

The problems are defined for any demand area and any demand distribution. However, most of the analysis in this paper pertains to uniform demand and a square area. It can be generalized to any convex polygonal shape and non-uniform demand. Finding the Voronoi diagram inside a polygonal area is quite easy. If demand is not uniform, calculating the

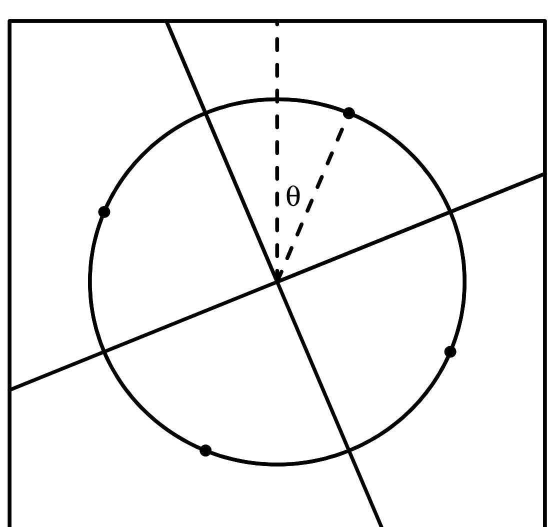

Fig. 1. EL and MER solutions.