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Discrete Optimization

Single machine scheduling to minimize total weighted earliness subject to minimal number of tardy jobs

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Abstract

Motivated by just-in-time manufacturing, we consider a single machine scheduling problem with dual criteria, i.e., the minimization of the total weighted earliness subject to minimum number of tardy jobs. We discuss several dominance properties of optimal solutions. We then develop a heuristic algorithm with time complexity $O(n^3)$ and a branch and bound algorithm to solve the problem. The computational experiments show that the heuristic algorithm is effective in terms of solution quality in many instances while the branch and bound algorithm is efficient for medium-size problems.

Keywords: Scheduling; Single machine; Dual criteria; Earliness; Number of tardy jobs

1. Introduction

Scheduling models with both earliness and tardiness costs are consistent with the philosophy of just-in-time (JIT) manufacturing, which emphasizes producing goods only when they are needed. In these scheduling models, jobs are scheduled to complete as close as possible to their due dates. A large body of literature on scheduling models with earliness and tardiness has appeared in the last two decades; see Baker and Scudder (1990) for an extensive review in 1990. However, most of the research focuses on JIT scheduling models with objective of minimizing total (weighted) costs of early and tardy jobs. For example, Sridharan and Zhou (1996) considered a single machine total weighted earliness and tardiness scheduling problem and developed a solution procedure based on decision theory; Cai and Zhou (1999) studied a parallel machine stochastic scheduling problem to minimize expected total cost for early and tardy jobs; Mazzini and Armentano (2001) studied a general single machine scheduling problem with early and tardy costs and developed a heuristic; Wan and Yen (2002) studied a general single machine scheduling problem with distinct due windows and weighted earliness and tardiness costs, and developed a tabu search procedure; Hino et al. (2005) developed a tabu search heuristic and a genetic algorithm to solve a single machine scheduling problem with a common due date to minimize total weighted earliness and tardiness costs.

On the other hand, only a few studies are concerned with JIT scheduling models with dual criteria. For single machine dual criteria scheduling models, Lee and Vairaktarakis (1993) provided a framework with classification and complexity results. Chen and Bulfin (1993) studied complexity of various single machine multi-criteria scheduling problems. Nagar et al. (1995) provided a detailed literature survey of multiple and bicriteria scheduling problems. Yen and Wan (2003) presented an extensive review of single machine bicriteria scheduling models, especially models with costs of early and tardy jobs. Vairaktarakis and Lee

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(1995) and Duffuaa et al. (1997) studied a single machine scheduling problem to minimize total tardiness subject to minimal number of tardy jobs, independently. They derived several dominance properties of optimal solutions, and developed heuristics as well as branch and bound algorithms that take advantage of the properties to solve the problem. For bicriteria scheduling models related to early and tardy costs, Chen et al. (1997) considered a single machine scheduling problem of minimizing total weighted earliness subject to maximum tardiness. They developed a heuristic and branch and bound algorithms based on the properties they derived to solve the problem. Chand and Schneeberger (1988) studied a single machine problem to minimize total weighted earliness subject to no tardy jobs. They transformed the problem into a scheduling problem of minimizing total weighted completion time, and developed heuristics and a dynamic programming procedure for solving the problem. However, their dynamic programming procedure is not efficient for problems of reasonable size as noted by the authors. Guner et al. (1998) considered one machine scheduling to minimize the maximum earliness with minimum number of tardy jobs. They proposed a procedure to minimize the maximum earliness when the set of tardy jobs is specified, and a branch and bound algorithm to minimize the maximum earliness subject to minimum number of tardy jobs. Karasakal and Koksalan (2000) developed a simulated annealing approach to two single machine bicriteria scheduling problems: one to minimize total flowtime and maximum earliness while the other to minimize total flowtime and number of tardy jobs. Later, Koksalan and Keha (2003) also developed genetic algorithms for the two problems.

Recently, Azizoglu et al. (2003a) considered a single machine scheduling problem with maximum earliness and number of tardy and no inserted idle time. They developed procedures to solve the problem optimally. Azizoglu et al. (2003b) further studied the same problem, but with allowed idle time. They gave polynomial time algorithms for problems of minimizing the number of tardy jobs subject to minimum maximum earliness, and of minimizing the maximum earliness subject to no tardy jobs and a given set of tardy jobs, respectively. Pathumnakul and Egbelu (2005) studied the same problem as in Chand and Schneeberger (1988) and developed a heuristic algorithm based on local optimality conditions. They showed numerically that their heuristic algorithm outperforms the methods by Chand and Schneeberger (1988).

In this paper, we consider a single machine bicriteria scheduling problem of minimizing the total weighted earliness subject to minimum number of tardy jobs, which is an extension of the problem studied in Chand and Schneeberger (1988) and Pathumnakul and Egbelu (2005). There is a couple of reasons for considering such dual criteria. In many practical situations, it is required to guarantee that as many jobs as possible to meet their due dates (i.e., to minimize the number of tardy jobs) since in such cases, having a job missing its due date is very costly. Thus minimization of the number of tardy jobs should be the primary concern. On the other hand, it is desirable to minimize the job earliness so as to minimize the inventory cost. A typical example is fashion manufacturing. In fashion manufacturing, if a job is overdue (on the basis of fashion season), then it misses the sale opportunity regardless of how overdue it is, which is very costly. Hence the minimization of the number of tardy jobs is the primary concern. Given this constraint, it is desirable to minimize the total weighted earliness so as to minimize the inventory cost.

The remainder of this paper is organized as follows. In Section 2, we give some notations and discuss several basic properties of optimal solutions of the problem. Section 3 is devoted to the development of algorithms for solving the problem, including a heuristic algorithm and a branch and bound algorithm. These algorithms are based on the properties developed in Section 2. Section 4 presents the computational results for the heuristic algorithm as well as the branch and bound algorithm. Section 5 concludes the paper.

2. Notations and dominance properties

Consider a scheduling problem with n jobs to be processed on one machine with the following assumptions:

(1) all the jobs are available at time zero;
(2) the machine can process at most one job at a time;
(3) no preemption is allowed; and,
(4) associated with each job j $(j = 1, 2, \ldots, n)$ there are a processing time p_j and a due date d_j.

The objective of the problem is to find a schedule that minimizes the total weighted earliness subject to the minimization of the number of tardy jobs. We denote this dual criteria as $\sum_j \alpha_j E_j | \sum_j U_j = U_{\min}$, where α_j is the earliness weight of job j $(j = 1, 2, \ldots, n)$, and $U_{\min}$ is the minimal number of tardy jobs.

Given a schedule π of the problem, we define the following notations.

$C_j(\pi)$ completion time of job j in schedule π;
$s_j(\pi)$ starting time of job j in schedule π;
$U_j(\pi)$ $\begin{cases} 1, & \text{if job } j \text{ is tardy,} \\ 0, & \text{otherwise.} \end{cases}$
$E_j(\pi)$ $\max\{0, d_j - C_j(\pi)\}$: earliness of job j in schedule π.

In cases where the schedule π is clearly known, we may not explicitly express the schedule π in the notations.

Based on the above notations, we have the following mathematical formulation for the scheduling problem:

$$\text{Min} \quad \sum_j \alpha_j E_j(\pi),$$

$$\text{Subject to:} \quad \sum_j U_j(\pi) = U_{\min} \quad \text{and} \quad \pi \in \Pi,$$

where Π denotes the set of all permutations of $\{1, 2, \ldots, n\}$. We denote this problem as (**P**). From Chand and Schnee-