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Discrete Optimization

Two phase algorithms for the bi-objective assignment problem

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Abstract

In this paper, we present several algorithms for the bi-objective assignment problem. The algorithms are based on the two phase method, which is a general technique to solve multi-objective combinatorial optimisation (MOCO) problems.

We give a description of the original two phase method for the bi-objective assignment problem, including an implementation of the variable fixing strategy of the original method. We propose several enhancements for the second phase, i.e., improved upper bounds and a combination of the two phase method with a population based heuristic using path relinking to improve computational performance. Finally, we describe a new technique for the second phase with a ranking approach, which outperforms all other tested algorithms.

All of the algorithms have been tested on instances of varying size and range of objective function coefficients. We discuss the results obtained and explain our observations based on the distribution of objective function values.
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1. The bi-objective assignment problem

In this paper, we investigate the exact solution of the bi-objective assignment problem. In Section 1, we introduce the problem, general definitions and a classification of efficient solutions. Our investigation is within the framework of the two phase method, originally proposed for the bi-objective assignment problem in Ulungu and Teghem (1995). Section 2 is mainly devoted to the description of that solution method, which we refer to as "the original two phase method". In Section 3, we present improvements of the original method and new algorithms for the second phase. These include the incorporation of a population based heuristic

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using path-relinking to obtain good starting solutions and the use of a ranking algorithm. All proposed algorithms are evaluated on a large set of numerical instances. Numerical results are reported and discussed in Section 4. The paper is concluded with a discussion on the distribution of objective values.

1.1. Problem formulation and characteristics

The single objective assignment problem (AP) is an integer programming problem that can be solved as a linear program due to total unimodularity of the constraint matrix. Efficient algorithms to solve it, e.g., the Hungarian method or the successive shortest paths method (Papadimitriou and Steiglitz, 1982; Ahuja et al., 1993) are well known.

In this paper, we consider the assignment problem with two objectives (BAP). It can be formulated as follows:

$$\begin{aligned} \min \quad & z_k(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^k x_{ij}, \quad k = 1,2, \\ & \sum_{i=1}^{n} x_{ij} = 1, \quad j = 1,\ldots,n \\ & \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1,\ldots,n \\ & x_{ij} \in \{0,1\}, \quad i,j = 1,\ldots,n, \end{aligned} \tag{BAP}$$

where all objective function coefficients c_{ij}^k are non-negative integers and $x = (x_{11},\ldots,x_{nn})$ is the matrix of decision variables.

Let X denote the set of all feasible solutions of (BAP). $X \subset \{0,1\}^{n^2} \subset \mathbb{R}^{n^2}$ is also called the feasible set in decision space. $Z = \{z(x) : x \in X\} \subset \mathbb{N}^2 \subset \mathbb{R}^2$ is called the feasible set in objective space.

In bi-objective optimisation, there is in general no feasible solution which minimises both objectives simultaneously.

Definition 1. A feasible solution $x^* \in X$ is called *efficient* if there does not exist any other feasible solution $x \in X$ such that $z_k(x) \leqslant z_k(x^*)$, $k = 1,2$, with at least one strict inequality. $z(x^*)$ is then called a *non-dominated point*. The set of efficient solutions is denoted by X_{E} and the image of X_{E} in Z is called the *non-dominated frontier* Z_{N}. If $x, x' \in X$ are such that $z_k(x) \leqslant z_k(x'), k = 1,2$, and $z(x) \neq z(x')$ we say that x dominates x' ($z(x)$ dominates $z(x')$).

The set of efficient solutions is partitioned in two subsets as follows.

- *Supported* efficient solutions are optimal solutions of a weighted sum single objective problem:

  $$\min\{\lambda_1 z_1(x) + \lambda_2 z_2(x) : x \in X\} \tag{BAP$_\lambda$}$$

  for some $\lambda_1, \lambda_2 > 0$. All supported non-dominated points are located on the "lower-left boundary" of the convex hull of Z (conv Z), i.e., they are non-dominated points of (conv Z) $+ \mathbb{R}^2_+$. By varying λ all supported solutions can be found. We use the notations X_{SE} and Z_{SN}, respectively, to denote supported efficient solutions and supported non-dominated points.
- *Non-supported* efficient solutions are efficient solutions that are not optimal solutions of (BAP)$_\lambda$ for any λ with $\lambda_1, \lambda_2 > 0$. Non-supported non-dominated points are located in the interior of (conv Z) $+ \mathbb{R}^2_+$. No theoretical characterisation which leads to an efficient computation of the non-supported efficient solutions is known. The sets of non-supported efficient solutions and non-supported non-dominated points are denoted X_{NE} and Z_{NN}, respectively.

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