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Interfaces with Other Disciplines

On the robustness of non-linear personalized price combinatorial auctions

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ABSTRACT

Though the VCG auction assumes a central place in the mechanism design literature, there are a number of reasons for favoring Iterative Combinatorial Auctions (ICAs). Several promising ICA formats were developed based on primal–dual and subgradient algorithms. Prices are interpreted as a feasible dual solution and the provisional allocation is interpreted as a feasible primal solution. iBundle(3), dVSV and Ascending Proxy Auction result in VCG payoffs when the coalitional value function satisfies buyer submodularity and bidders bid straightforward, which is an ex-post Nash equilibrium in this case. iBEA and CreditDebit auctions do not even require the buyer submodularity and achieve the same properties for general valuations. Often, however, one cannot assume straightforward bidding and it is not clear from the theory how these non-linear personalized price auctions (NLPPAs) perform in this case. Robustness of auctions with respect to different bidding behavior is a critical issue for any application. We conducted a large number of computational experiments to analyze the performance of NLPPAs with respect to different bidding strategies and valuation models. We compare NLPPAs with the VCG auction and with ICAs with linear prices, such as ALPS and the Combinatorial Clock Auction. While NLPPAs performed very well in case of straightforward bidding, we observe problems with revenue, efficiency, and speed of convergence when bidders deviate.

1. Introduction

"Experience in both the field and laboratory suggest that in complex economic environments iterative auctions [...] produce better results than sealed-bid auctions" (Porter et al., 2003). Several authors have tried to develop indirect auctions with strong incentive properties to overcome these problems, as in Iterative Combinatorial Auctions, bidders do not have to reveal all their true preferences in one round as would be necessary in Vickrey–Clarke–Groves (VCG) mechanisms (Parkes and Ungar, 2000; Ausubel and Milgrom, 2006b). Iterative Combinatorial Auctions (ICAs) are also strategically simpler than "first-price sealed-bid" auction designs (Vickrey, 1961).

The goal of achieving VCG payoffs in a CA is twofold. The VCG mechanism is the unique auction that has a dominant-strategy property, leads to efficient outcomes, and takes zero payment from losing bidders (Green and Laffont, 1977; Ausubel et al., 2006, p. 93). Though the VCG auction assumes a central place in the mechanism design literature, its results are outside of the core, when bidders are not substitutes (see Definition 5). If this is the case, the seller's revenue can be uncompetitively low, and opens up bidder non-monotonicity problems, and possibilities for collusion and shill-bidding (see Ausubel and Milgrom (2006b) and Day and Milgrom (2008) for a more detailed discussion). Bidder monotonicity means, that auctioneer revenue cannot decrease with additional bidders.

Two main approaches have been discussed in the literature for the design of ICAs. Some authors try to maintain linear prices at the expense of some levels in efficiency (Rassenti et al., 1982; Porter et al., 2003; Kwasnica et al., 2005; Bichler et al., 2009). For example, pseudo-dual prices describe an approach, where integer constraints of the winner determination problem are relaxed and linear prices are derived from a restricted dual problem. Another school of thought uses non-linear personalized ask prices. This means that there is an individual ask prices for each bundle and each bidder in the worst case. The approach is based on an extended linear program of the winner determination problem introduced by Bikhchandani and Ostroy (2002) that always implements an integral solution, even without integer restrictions on variables. Consequently, the non-linear and personalized ask prices derived from the dual variables will lead to competitive equilibrium, maximizing allocative efficiency when bidders follow a *straightforward* bidding strategy. Straightforward bidding describes a strategy in which, in each round, the bidder submits the minimum bid on the bundles maximizing his payoff at the current ask prices.

While the original formulation in Bikhchandani and Ostroy (2002) leads to an exponentially large number of additional variables, it has inspired a number of practical auction designs.

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Primal–dual algorithms and subgradient algorithms have been used as a conceptual framework to design iterative combinatorial auctions such as iBundle(3) (Parkes and Ungar, 2000), the Ascending Proxy Auction (Ausubel and Milgrom, 2006a), and dVSV (de Vries et al., 2007). All three auction designs result in VCG payments, when the *bidder submodularity condition* is satisfied and bidders follow a straightforward strategy. It can also be shown that there are incentives to do so: a straightforward bidding strategy is an ex-post equilibrium as long as the submodular valuations condition is satisfied (Mishra and Parkes, 2007). An ex-post equilibrium assumes, however, that all bidders bid straightforward, i.e., they play their best-response strategy. Given, that bidders do not know a priori, whether bidder submodularity conditions hold, this is a strong assumption.

Although, de Vries et al. (2007) show that for private valuation models without restrictions ascending combinatorial auctions cannot achieve VCG payoffs, newer approaches try to overcome this by extending the definition of ascending price auctions. For example, Ausubel (2006) uses multiple price paths. The CreditDebit auction by Mishra and Parkes (2007), based on the dVSV auction, calculates discounts on the quoted prices, and in a similar way, iBEA is based on iBundle(3). These formats will terminate with VCG payments for general valuations. In the following, we will call iBundle, the Ascending Proxy Auction, dVSV, CreditDebit, and iBEA *non-linear personalized price auction designs (NLPPAs)*.

Clearly, NLPPAs can be considered a fundamental contribution to the combinatorial auction theory, as they describe iterative auction designs that are fully efficient. However, they are based on a number of assumptions. In particular, straightforward bidding might not hold in practical settings where bidders have bounded rationality, given that bidders do not know, whether the submodularity condition holds, and there is a huge number of bundles a bidder has to deal with. Recent experimental work has actually shown that bidders did not follow a pure best-response strategy, even in simple settings with only a few items (Scheffel et al., in press). Therefore, it is important to understand their performance in case of non-straightforward bidding strategies, when bidders either cannot follow such a strategy for computational or cognitive reasons, or deliberately choose another strategy.

One of the beauties of double auction markets is their robustness against simple, even random bidding strategies, as shown by Gode and Sunder (1993). Similarly, we will introduce the notion of *robustness of combinatorial auctions*, which refers to average efficiency they achieve with respect to different bidding strategies. The fundamental question of this paper is: "*How robust are NLPPAs against non-straightforward bidding strategies*." Any evidence about the performance of auction designs with non-straightforward bidding strategies will not only be important for practical applications, it should also provide a basis for the development of efficient and robust auction designs in the future.

In the following, we will describe the results of computational experiments analyzing allocative efficiency, revenue distribution, ability to address the threshold problem, and speed of convergence of different NLPPAs against those of the VCG auction, ALPS (Approximative Linear PriceS), and the Combinatorial Clock Auction, two designs that use linear-prices based on different value models and different bidding strategies. We have decided to run our experiments without activity rules, because several rules have been discussed and using a specific one will distort the comparison. The often used monotonicity rule for example makes it impossible to follow a straightforward bidding strategy.

The paper is structured as follows. In Section 2, we will briefly summarize the economic environment, the auction designs in question, and the performance metrics that we use. Section 3 will describe the setup of our computational experiments. In Section 4, we will summarize the results and then provide conclusions in Section 5.

2. Related theory and auction formats

This section provides an overview of iterative combinatorial auctions and the relevant theory so that the paper is self-contained. We refer the reader to Parkes (2006) for a more detailed introduction to ICAs.

2.1. Winner determination and pricing

The typical bidding process in an ICA consists of the steps of bid submission, bid evaluation (aka winner determination, market clearing, or resource allocation) followed by feedback to the bidders. The feedback is typically given in form of ask prices and the provisional allocation.

Let $\mathcal{K} = \{1,\ldots,m\}$ denote the set of items indexed by k and $\mathcal{I} = \{1,\ldots,n\}$ denote the set of bidders indexed by i with private valuations $v_i(S) \geqslant 0$, $v_i(\emptyset) = 0$ for bundles $S \subseteq \mathcal{K}$. In addition we assume free disposal: If $S \subset T$ then $v_i(S) \leqslant v_i(T)$.

Given the private bidder valuations for all possible bundles, the efficient allocation can be found by solving the *Winner Determination Problem (WDP)*. WDP can be formulated as a binary program using the decision variables $x_i(S)$ which indicate whether the bid of the bidder i for the bundle S belongs to the allocation:

$$\begin{aligned} &\max_{x_i(S)} \sum_{S\subseteq\mathcal{K}} \sum_{i\in\mathcal{I}} x_i(S)\, v_i(S), \\ &\text{s.t.} \\ &\quad \sum_{S\subseteq\mathcal{K}} x_i(S) \leqslant 1 \quad \forall i \in \mathcal{I}, \\ &\quad \sum_{S:k\in S} \sum_{i\in\mathcal{I}} x_i(S) \leqslant 1 \quad \forall k \in \mathcal{K}, \\ &\quad x_i(S) \in \{0,1\} \quad \forall i, S, \end{aligned} \tag{WDP}$$

The first set of constraints guarantees that any bidder can win at most one bundle, which is only relevant for the XOR bidding language. The XOR language is used because it is fully expressive compared to the OR language which allows a bidder to win more than one bid. Subadditive valuations, where a bundle is worth less than the sum of individual items, cannot be expressed using the OR bidding language. The second set of constraints ensures that each item is only allocated once. Much research has focused on solving the winner determination problem, which is known to be NP-hard (Rothkopf et al., 1998; Sandholm, 1999; Park and Rothkopf, 2005).

Having determined the winning bids, the auctioneer needs to decide what the winners should pay. A simple approach is for bidders to pay the amount of their bids. However, this creates incentives for bidders to shade their bids and might ultimatively lead to strategic complexity, i.e., to speculation and inefficient allocations.

2.2. Vickrey prices and competitive equilibrium prices

The VCG auction is a generalization of the Vickrey auction for multiple heterogeneous goods. In this auction bidders have a dominant strategy of reporting their true valuations $v_i(S)$ on all bundles S to the auctioneer, who then determines the allocation and respective Vickrey prices. The VCG design charges the bidders the opportunity costs of the items they win, rather than their bid prices.

Although it has a simple dominant strategy, VCG design suffers from a number of practical problems since its outcome can be outside of the *core* (Rothkopf, 2007; Ausubel and Milgrom, 2006b).

Formally, let N denote the set of all bidders $\mathcal{I}$ and the auctioneer with $i \in N$, and $M \subseteq N$ be a coalition of bidders with the auctioneer. Let $w(M)$ denote the coalitional value for a subset M, equal to the value of the WDP with all bidders $i \in M$ involved. (N, w) is the