Available online at www.sciencedirect.com
ScienceDirect
European Journal of Operational Research 181 (2007) 1097–1111
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

A Mizuno–Todd–Ye type predictor–corrector algorithm for sufficient linear complementarity problems

Tibor Illés *, Marianna Nagy

Department of Operational Research, Eötvös Loránd University of Sciences, Pázmány Péter sétány 1/C, 1117 Budapest, Hungary

Received 5 October 2004; accepted 10 August 2005
Available online 2 May 2006

Abstract

We analyze a version of the Mizuno–Todd–Ye predictor–corrector interior point algorithm for the $\mathscr{P}_*(\kappa)$-matrix linear complementarity problem (LCP). We assume the existence of a strictly positive feasible solution. Our version of the Mizuno–Todd–Ye predictor–corrector algorithm is a generalization of Potra's [F.A. Potra, The Mizuno–Todd–Ye algorithm in a larger neighborhood of the central path, European Journal of Operational Research 143 (2002) 257–267] results on the LCP with $\mathscr{P}_*(\kappa)$-matrices. We are using a $\|\mathbf{v}^{-1}-\mathbf{v}\|$ proximity measure like Potra to derive iteration complexity result for this algorithm . Our algorithm is different from Miao's method [J. Miao, A quadratically convergent $\mathrm{O}\big((\kappa+1)\sqrt{n}L\big)$-iteration algorithm for the $P_*(\kappa)$-matrix linear complementarity problem, Mathematical Programming 69 (1995) 355–368] in both the proximity measure used and the way of updating the centrality parameter. Our analysis is easier than the previously stated results. We also show that the iteration complexity of our algorithm is $\mathrm{O}\big((1+\kappa)^{\frac{3}{2}}\sqrt{n}L\big)$.

Keywords: Linear complementarity problem; Sufficient matrix; $\mathscr{P}_*(\kappa)$-matrix; Interior point method; Mizuno–Todd–Ye predictor–corrector algorithm

1. Introduction

Consider the *linear complementarity problem* (LCP): find vectors $\mathbf{x},\mathbf{s}\in\mathbb{R}^n$, which satisfy the constraints

$$-M\mathbf{x}+\mathbf{s}=\mathbf{q},\quad \mathbf{x}\mathbf{s}=\mathbf{0},\ \ \mathbf{x},\mathbf{s}\geqslant\mathbf{0}, \tag{1}$$

where $M\in\mathbb{R}^{n\times n}$ and $\mathbf{q}\in\mathbb{R}^n$.

The linear complementarity problem belongs to the class of $\mathbb{NP}$-complete problems, since the feasibility problem of linear equations with binary variables can be described as an LCP problem [6]. Therefore, we cannot expect an efficient solution method for the linear complementarity problem without special property of the matrix M.

* Corresponding author. Tel.: +36 1 2090555; fax: +36 1 3812156.
E-mail addresses: illes@math.elte.hu (T. Illés), nmariann@cs.elte.hu (M. Nagy).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2005.08.031

We assume that the matrix M is a $\mathscr{P}_*(\kappa)$-matrix and we generalize the well-known Mizuno–Todd–Ye predictor–corrector interior point algorithm for this class of the linear complementarity problems.

One of the first version of the predictor–corrector interior point algorithm for linear programming problems was initiated by Sonnevend et al. [16]. This algorithm needs more corrector steps after each predictor step in order to return to the appropriate neighborhood of the central path. Mizuno et al. [8] published such a predictor–corrector interior-point method for the linear programming problem in which each predictor step is followed by a single corrector step and whose iteration complexity is the best known in the linear programming literature. Anstreicher and Ye [18] extended this result to the linear complementarity problem with a positive semidefinite matrix with the same iteration complexity.

In one of the best papers on interior-point algorithms, Kojima et al. [5] offered a polynomial primal-dual interior point method for the positive semidefinite matrix linear complementarity problem. The properties of a more general matrix class can be formulated in a natural way from the iteration complexity analysis of their algorithm. This class is called a $\mathscr{P}_*(\kappa)$-matrix by Kojima et al. [6]. The primal-dual interior point algorithm of Kojima et al. is generalized to $\mathscr{P}_*(\kappa)$-matrices. This algorithm is also polynomial if a $\hat{\kappa} \geqslant 0$ is known, such that the matrix of the problem is $\mathscr{P}_*(\hat{\kappa})$-matrix. The iteration complexity is a polynomial of $\hat{\kappa}$, the dimension n and the bit length L of the problem.

Since Kojima et al. published their book [6] on interior point methods for LCPs, the quality of a variant of an interior point algorithm is measured by the fact whether it can be generalized to the $\mathscr{P}_*(\kappa)$-matrix linear complementarity problem or not.

The natural outcome of this was the emergence of different interior-point algorithms for the $\mathscr{P}_*(\kappa)$-matrix linear complementarity problem in the mid-90s.

Several variants of the Mizuno–Todd–Ye type predictor–corrector interior-point algorithm are known in the literature. First Miao [7], later Potra and Sheng [11] gave a generalization of Mizuno–Todd–Ye predictor–corrector algorithm with the $\mathscr{P}_*(\kappa)$-matrix linear complementarity problem assuming the existence of a strictly positive solution. Miao updated the central path parameter μ in such a way that $\mathbf{x}^{\mathrm{T}}\mathbf{s}/n = \mu$ equality holds throughout. Therefore, the updating of μ is more complicated than in the skew-symmetric case, where $\mu' = (1-\alpha)\mu$ and α is the length of the Newton-step in the predictor phase. Further generalization has been established: Ji et al. [4] extended the algorithm to the infeasible linear complementarity problem, Potra and Sheng [12,13] to the infeasible and degenerate problem. In these methods the parameter μ is updated by $\mu' = (1-\alpha)\mu$, thus $\mathbf{x}^{\mathrm{T}}\mathbf{s}/n \neq \mu$ in their cases.

The Mizuno–Todd–Ye type predictor–corrector algorithm for the skew-symmetric or positive semidefinite linear complementarity problem (horizontal linear complementarity problem, HLCP) of [14] is the basis of this paper. As it is explained in [14] the Mizuno–Todd–Ye predictor–corrector method based on a very simple and elegant idea that is used in various other fields of computational mathematics such as the numerical methods of differential equations and continuation methods. We already mentioned that there exists $\mathbb{NP}$-complete linear complementarity problems. The $\mathbb{NP}$-completeness of linear complementarity problems are related to the properties of the matrix M. Therefore, our aim is to generalize the Mizuno–Todd–Ye algorithm – which is one of the most remarkable interior point method for linear programming and quadratic programming – for the widest possible matrix class where the method is polynomial. This is the $\mathscr{P}_*$ matrix class defined by Kojima et al. [6].

At choosing the proximity measure we followed Potra ($\|\mathbf{v}^{-1} - \mathbf{v}\|$) in contrast to the previous works ($\|\mathbf{v} - \mathbf{e}\|$, where $\mathbf{v} = \sqrt{\mathbf{xs}/\mu}$). The reason was that in practice interior point algorithms make longer step in wide neighborhood, therefore their practical performance might even be better than the theoretical one. Furthermore Mizuno–Todd–Ye algorithm for $\mathscr{P}_*(\kappa)$ linear complementarity problems was not generalized earlier for large neighborhoods.

Summarizing the previous, we may state, that we present a new variant of the Mizuno–Todd–Ye predictor–corrector algorithm for $\mathscr{P}_*(\kappa)$ linear complementarity problems that uses self-regular proximity measure $\|\mathbf{v}^{-1} - \mathbf{v}\|$, and therefore the iterates lies in a wider neighborhood of the central path than in the earlier published Mizuno–Todd–Ye type algorithms for this class of problems. Our algorithm's iteration complexity is $\mathrm{O}\big((1+\kappa)^{\frac{3}{2}}\sqrt{n}L\big)$.

The following section deals with the fundamental properties of $\mathscr{P}_*(\kappa)$-matrices and with some well-known results. Section 3 describes the predictor–corrector algorithm, and the following sections analyze the method. Section 3.2 deals with the predictor step and determines the length of the Newton-step. The next part examines