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Algorithms and the calculation of Nash equilibria for multi-objective control of time-discrete systems and polynomial-time algorithms for dynamic c-games on networks

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Abstract

We consider a multi-objective control problem of time-discrete systems with given starting and final states. The dynamics of the system are controlled by p actors (players). Each of the players intends to minimize his own integral-time cost of the system's transitions using a certain admissible trajectory. Nash Equilibria conditions are derived and algorithms for solving dynamic games in positional form are proposed in this paper. The existence theorem for Nash equilibria is related to the introduction of an auxiliary dynamic c-game. Stationary and non-stationary cases are described. The paper concludes with a complexity analysis for that decision process.

Keywords: Time-discrete system; Multi-objective control; c-game on networks; Nash equilibria; Pareto optima; Multiobjective games; Pareto–Nash equilibria

1. The general model

Let L be a discrete dynamic system with the set of states $X \subseteq \mathbb{R}^m$ (see [1,3,4,12]). At every time-step $t = 0, 1, 2 \ldots$ the state of L is $x(t) \in X$. Two states x_0 and x_f in X are given, where $x_0 = x(0)$ represents the starting point of L and x_f is the state into which the system L must be brought, i.e. x_f is the final state of L. We assume that the system L reaches the final state x_f at the time step $T(x_f)$ such that

$$T_1 \leqslant T(x_f) \leqslant T_2,$$

where T_1 and T_2 are given. The dynamics of the system are described as follows:

$$x(t+1) = g_t(x(t), u(t)), \quad t = 0, 1, 2, \ldots$$

with $x(0) = x_0$ and where $u(t) \in \mathbb{R}^m$ represent the vector of control parameters.

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For $u(t)$, at each time step t let $U_t(x(t))$ be a given nonempty set such that

$$u(t) \in U_t(x(t)), \quad t = 0, 1, 2, \ldots \tag{1}$$

Here $U_t(x(t))$ is the admissible (decision) set for the vector of control parameters at the time-step t when the state of system L is $x = x(t) \in X$. We assume that the derivatives $g_t(x(t), u(t))$ are known and uniquely determine $x(t+1)$ for given $x(t)$ and $u(t)$ at every point in time $t = 0, 1, 2, \ldots$ In addition we assume that at each point in time t the cost $c_t(x(t), x(t+1))$ is known. Here $c_t(x(t), x(t+1))$ equals $c_t(x(t), g_t(x(t), u(t)))$ of the system's transition from the state $x(t)$ to the state $x(t+1)$:

Let

$$x_0 = x(0), x(1), x(2), \ldots, x(t), \ldots$$

be the trajectory generated by the given vectors of control parameters

$$u(0), u(1), \ldots, u(t-1), \ldots$$

Either this trajectory passes through the state x_f at time $T(x_f)$ or it does not pass through x_f. By

$$F_{x_0 x_f}(u(t)) = \sum_{t=0}^{T(x_f)-1} c_t(x(t), g_t(x(t), u(t))) \tag{2}$$

we denote the integral-time cost of system's transition from x_0 to x_f if $T_1 \leqslant T(x_f) \leqslant T_2$; otherwise we stipulate

$$F_{x_0 x_f}(u(t)) = \infty.$$

Remark 1 (*Single-Objective Control Problem*). The discrete control problem is defined as follows: Minimize the function $F_{x_0 x_f}(u(t))$ which is defined by (2) according to (1).

Thus, we consider different cases of the discrete control problem: In one case there is a with a fixed number of stages ($T_1 = T_2$), in the other case the number of stages is not limited ($T_1 = 1, T_2 = \infty$).

If we study the game version of the considered problem we have to assume, as before, that the dynamics of the time-discrete system are controlled by p actors (players) and each of them intends to minimize his own integral-time cost of system's transitions using a certain admissible trajectory. Such a class of problems is motivated by bargaining procedures within electricity markets. These are described in [23]. The problem formulation is also contained in a slightly modified version in [18]. In [18] k-partite networks are considered.

2. Problem formulation for determining a Nash equilibrium – the decision procedure

We now consider the following multi-objective control problem for time-discrete systems:

Consider the dynamic system L over discrete moments in time $t = 0, 1, 2, \ldots$ At each time-step t the state of L is $x(t) \in X \subseteq \mathbb{R}^m$. The dynamics of the system L are controlled by p players and are described as follows:

$$x(t+1) = g_t(x(t), u^1(t), u^2(t), \ldots, u^p(t)), \quad t = 0, 1, 2, \ldots \tag{3}$$

Here $x(0) = x_0$ is the starting point of the system L and $u^i(t) \in \mathbb{R}^{m_i}$ represents the vector of control parameters of player i, $i \in \{1, 2, \ldots, p\}$. The state $x(t+1)$ of the system L at time-step $t+1$ can be obtained uniquely if the state $x(t)$ at the time-step t is known and the players $1, 2, \ldots, p$ fix their vectors of control parameters $u^1(t), u^2(t), \ldots, u^p(t)$, respectively. For each player i, $i \in \{1, 2, \ldots, p\}$ the admissible sets $U_t^i(x(t))$ for the vectors of control parameters $u^i(t)$ are given, i.e.

$$u^i(t) \in U_t^i(x(t)), \qquad t = 0, 1, 2, \ldots; \quad i = \overline{1, p}.$$

We assume that $U_t^i(x(t+1)),\ t = 0, 1, 2, \ldots; \quad i = \overline{1, p}$, are non-empty finite sets and that

$$U_t^i(x(t)) \cap U_t^j(x(t)) = \emptyset, \quad i \neq j, \quad t = 0, 1, 2, \ldots$$

We assume that the players $1, 2, \ldots, p$ fix their vectors of control parameters

$$u^1(t), u^2(t), \ldots, u^p(t); \quad t = 0, 1, 2, \ldots,$$