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Portfolio selection under strict uncertainty: A multi-criteria methodology and its application to the Frankfurt and Vienna Stock Exchanges

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Abstract

In modern portfolio theory, it is common practice to first compute the risk-reward efficient frontier and then to support an individual investor in selecting a portfolio that meets his/her preferences for profitability and risk. Potential flaws include (a) the assumption that past data provide sufficient evidence for predicting the future performances of the securities under consideration and (b) the necessity to mathematically determine or approximate the investor's utility function. In this paper, we propose a methodology whose initial phase filters portfolios that are inefficient from a historical perspective. While this is consistent with traditional approaches, the second phase differs from the standard approach as it uses a decision table constructed by considering multiple scenarios assuming strict uncertainty. The table cells measure consequences by a multi-criteria linear performance index of simulated future returns, which avoids difficulties with performance ratios. The real world applicability is illustrated through two studies based on data from the stock exchanges in Frankfurt and Vienna.

Keywords: Multi-criteria performance indices; Portfolio analysis; Simulation; Strict uncertainty

1. Introduction

Identifying the "best" portfolio of assets for an individual investor is one of the principal challenges in the world of finance. Based on the mean–variance (E-V) model of portfolio selection by Markowitz (1952, 1991) and its utilization for a capital market model (CAPM) by Tobin (1958), numerous researchers have contributed to the development of modern portfolio theory (cf. Constantinides and Malliaris, 1995; Elton and Gruber, 1999). As a result, nowadays it has become common practice to extend the classical economic model of financial investment to multi-criteria decision making for the purpose of supporting large-scale investors in setting up their portfolios with respect to (i) their

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preferences for profitability and risk and (ii) the uncertain development of stock markets.

Finance has traditionally recognized the necessity to compute the risk-reward efficient frontier. However, the subsequent task of determining the utility function for an individual investor in order to select the most attractive "point" on that frontier has remained a critical issue. Accordingly, several approaches to multiple criteria decision making aim at eliciting the investor's preferences and/or at proposing appropriate portfolios (for an overview cf. Steuer and Na, 2003). To this end, Ballestero and Romero (1996) introduced a compromise programming model (cf. Zeleny, 1982; Yu, 1985) to bound the "average" investor's utility optimum between two close points on the efficient frontier. The underlying theorem was subsequently modified to approximate an individual investor's optimal utility (Ballestero, 1998). This technique performed remarkably well for the common class of bi-attribute utility functions characterized by expected return and risk (or more precisely, an index of profitability and an index of safety). In contrast to attempts that simply apply standard methods of multi-criteria decision making to portfolio selection, Ballestero's utility bounding approach is based on the strict principles of economics and finance.

Unlike other studies in which bounding plays an essential role, this paper does not apply the bounding stage. Let us explain and motivate our new approach to portfolio selection that has not been published as yet; however, note that its mathematical foundation has been introduced by Ballestero (2002) and is useful not only in finance but in many other fields as well. This paper emphasizes portfolio selection based on future performance simulated under strict uncertainty instead of emphasizing choice entirely based on past data. While there are investors who absolutely believe in the predictive ability of historical information and also investors who completely refuse that idea, our paper is intended for neither of them, but for investors in-between. These investors realize that past data should not be ignored when necessary but used in a limited way. Determining the E-V efficient frontier from historical information is a necessary preliminary step to filter inefficient portfolios that otherwise would considerably enlarge and thus complicate the decision table under strict uncertainty. Even for investors reluctant to rely on past data, this may be a major reason to use them. In the proposed selection process, the first phase is aimed to obtain the efficient frontier while in the second phase uncertainty from the stock market future development is taken into account. To this end, the efficient stock portfolios can be combined with risk-free assets in the second phase. A decision table is then set up with several potential scenarios of the market. In this table, each cell represents the simulated future performance of the ith pre-selected efficient portfolio (possibly combined with risk-free assets) when the jth scenario becomes true. As a means of measuring performance, we use a multi-criteria linear index reflecting the investor's preferences for profitability and safety. This measure has advantages over standard performance ratios as (i) our performance index is a multi-criteria utility function while a performance ratio is not and (ii) no problem with zero denominators arises from the multi-criteria index. Through the decision table, the pre-selected efficient portfolios and their blends with risk-free assets are ranked. In short, the proposed approach provides a well-founded and straightforward methodology to identify the particular investment portfolio that best fits the notions of a pragmatic investor faced with an uncertain market. Note that pragmatism means neither working without past data – essential for the classic paradigm (Sharpe, 1997) – nor neglecting future market scenarios.

The remainder of this paper is organized as follows. Section 2 provides background information on modern portfolio theory, while Section 3 focuses on the proposed multi-criteria portfolio selection methodology. By presenting two studies based on real data from the Frankfurt and Viennese stock markets, Section 4 starts out with a brief market description and then gives a step-by-step instruction of how an investor is guided towards his/her favorite portfolio and how that portfolio is constructed. Finally, Section 5 provides concluding remarks.

2. Background

Modern portfolio theory is based on (i) analyzing risk by focusing on the investor's portfolio instead of individual securities, and (ii) determining and exploiting the E-V efficient frontier, namely, minimizing risk (commonly measured in terms of variance) for every level of expected return. The latter has its analytical foundation in Von Neumann and Morgenstern's (1944) utility theory under uncertainty. In the E-V framework, this theory implies the following assumptions: (i) risk aversion (Arrow,