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Extensions to STaTS for practical applications of the facility layout problem

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ABSTRACT

We consider a very general case of the facility layout problem, which allows incorporating various aspects appearing in real life applications. These aspects include loose requirements on facilities' footprints, each of which only needs to be of rectangular shape and can optionally be restricted concerning the surface area or the aspect ratio. Compared to former approaches other generalizations of practical relevance are multiple, not necessarily rectangular workshops, exclusion zones in workshops, predefined positions of facilities, the consideration of aisles, and the adherence of further restrictions such as the enforced placement of certain facilities next to an exterior wall or a minimum distance between certain pairs of facilities. Although different objectives could be applied, we especially focus on the most relevant one in practice, the minimization of transportation costs.

We show that this problem can heuristically be solved using an extension of the Slicing Tree and Tabu Search (STaTS) based approach. The application of this algorithm on practical data shows its effectiveness. The paper concludes with a step-by-step guide for the application of STaTS in practice.

1. Introduction and problem description

1.1. The facility layout problem

A facility layout problem (FLP) is concerned with the question of positioning facilities to locations. This very general problem is usually specified in different ways. In the past many approaches for the numerous variants of layout planning have been developed. Due to the complexity and peculiarities of the problem, many of these approaches are based on simplifying assumptions. However, a large number of practical applications does not fulfill all assumptions proposed by the various models.

In order to overcome the disadvantages of a too specified model, let us propose a more general framework of a FLP, representing a larger number of practical specifications at the cost of the simplicity of the model.

There are n rectangular facilities each of which with given surface area and a given set of allowed shapes. These facilities are to be positioned in one out of several right-angled workshops with given dimensions so that no two facilities overlap. Alternatively the dimension of a workshop can also be unrestricted which leads to an arrangement of facilities relative to each other.

This framework requires not only deciding on the position of the facilities, but also on their exact shape. Let us formalize this problem using the Euclidean plane $\mathbb{R}^2$ with the Cartesian coordinate system. The abscissa will be denoted by x and the ordinate by y. Then the following input data has to be given for the facility layout problem.

- A closed, not necessarily connected set $W \subseteq \mathbb{R}^2$ describing the available space for placing facilities. W represents the workshop(s). As mentioned above, we assume each workshop to be of right-angled shape.The restriction of right-angled workshop shapes is not very restrictive as any closed subset of $\mathbb{R}^2$ can be approximated arbitrarily accurate by right-angled areas. However, for practical applications the number of "corners" should not exceed a certain number, say 10. So prevalent shapes such as rectangles, L-shapes, or Z-shapes are allowed.
- n facilities, each of which has a given surface area $a(i) \in \mathbb{R}$, for all $i \in \{1, 2, \ldots, n\}$.
- For each facility i, there is a set $S(i)$ of (rectangular) shapes that can be adopted by this facility. These shapes can be denoted by the potential positions of i's top right corner relative to the bottom left corner. That means, under the assumption that the bottom left corner is placed on the origin $(0,0)$, each element of $S(i)$ describes an allowed position of the top right corner, i.e. $S(i) \subseteq \{(x,y) \in \mathbb{R}^2 | x \cdot y = a(i)\}$

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– Optionally, there might be further restrictions concerning the placement of facilities or an objective function (both to be described below).

Given these input data, the FLP is concerned with the question of finding a position and a concrete shape for each facility. The position of the facility can be characterized by the position of its bottom left corner, denoted by $p^i = \left(p_x^i, p_y^i\right) \in \mathbb{R}^2$. The exact shape is then determined by the position of the top right corner (in relation to the bottom left corner), denoted by $s^i = \left(s_x^i, s_y^i\right) \in S(i)$. The expanse covered by a facility is given by $E(i) := \{(x,y) \in \mathbb{R}^2 | p_x^i \leqslant x < p_x^i + s_x^i,\ p_y^i \leqslant y < p_y^i + s_y^i\}$. The FLP is now to find a mapping

$$\begin{aligned} &\phi : \{1,2,\ldots,n\} \rightarrow \mathbb{R}^2 \times \mathbb{R}^2, \\ &\phi(i) \mapsto (p^i, s^i), \\ \text{s.t.}\quad &E(i) \subseteq W \quad \forall i \in \{1,2,\ldots,n\}, \\ &E(i) \cap E(j) = \emptyset \quad \forall i \neq j, \quad i,j \in \{1,2,\ldots,n\}. \end{aligned}$$

The first restriction ensures that the facilities are placed within the workshops, whereas the second restriction prohibits overlapping facilities.

In case of unlimited workshop space, finding a feasible solution is easy. However, even if the available space in the workshop(s) considerably exceeds the sum of the required space of all facilities, a feasible solution might not exist, or might be hard to obtain.

Several objectives have been formulated for the FLP. Among the most common ones are the minimization of transportation costs between facilities or the minimization of the smallest rectangle containing all facilities (especially if $W = \mathbb{R}^2$). We will focus on the minimization of transportation costs as follows. For each ordered pair (i,j) of facilities, a certain flow $f_{ij} \in \mathbb{R}$ and material transportation costs per unit and distance $c_{ij} \in \mathbb{R}$ are given. Furthermore a metric $d : \mathbb{R}^2 \times \mathbb{R}^2 \rightarrow \mathbb{R}$ is given, which allows us to determine the distance d_{ij} between two facilities i and j by means of the distance of their central points: $d_{ij} := d\left(\left(s_x^i + \frac{p_x^i}{2}, s_y^i + \frac{p_y^i}{2}\right), \left(s_x^j + \frac{p_x^j}{2}, s_y^j + \frac{p_y^j}{2}\right)\right)$. Note that d_{ij} itself defines a pseudo-metric on the set of connected subsets of $\mathbb{R}^2$. Using these data, the objective of the FLP is

$$\min \quad F = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} d_{ij} \cdot f_{ij} \cdot c_{ij}.$$

To the very best of our knowledge, no paper has been presented which proposes a solution method for this general FLP. For $W = \mathbb{R}^2$, the Slicing Tree and Tabu Search (STaTS) based approach of Scholz et al. (2009) can be applied. We will extend this approach in order to cope with right-angled workshops and in order to consider the following additional restrictions that might appear in real life situations.

1. Some facilities must be placed in certain areas of the workshops (e.g. because of different ceiling heights, or an inventory must be placed at a certain point). If $W(i)$ describes the area in which facility i may be placed, we introduce the restriction
$$E(i) \subseteq W(i).$$
2. All facilities and workshop doors must be connected via aisles. As doors can be modeled as facilities with a fixed position, we can w.l.o.g. assume that only facilities have to be connected via aisles. This leads to the restriction
$$\forall i,j \in \{1,2,\ldots,n\} \exists \Delta \subseteq W \setminus \bigcup_k E(k) : E(i) \cup E(j) \cup \Delta \text{ is a connected space}.$$
For practical applications we might need to force that the aisles have a minimum width, i.e. only those Δ can be chosen that ensure the minimum width A (e.g. for forklifts).
3. Some facilities must be placed next to an exterior wall (e.g. because of an exhaust air conduit). Let $Wall \subseteq \{1,2,\ldots,n\}$ be the set of facilities to be placed next to a wall. Then we force
$$\inf \{d((x,y),(x',y')) | (x,y) \in E(i), (x',y') \notin W\} = 0 \quad \forall i \in Wall.$$
Note that this restriction makes only sense if W is bounded somewhere. We have used the same term d for the metric as in the objective function. However, it would be possible to apply different metrics.
4. For some pairs of facilities, a separation distance $d_{\min}$ has to be respected. A reason for that could be dust (oscillation) sensitive and dust raising (oscillation causing) machines. We consider this fact using the following restriction in which $BadNeighbors \subset \{1,2,\ldots,n\} \times \{1,2,\ldots,n\}$ describes all pairs of facilities with separation distance.
$$\min \{d((x,y),(x',y')) | (x,y) \in E(i), (x',y') \in E(j)\} \geqslant d_{\min} \quad \forall (i,j) \in BadNeighbors.$$

Put together, we are to solve the following optimization problem:

$$\begin{aligned} \min_{\phi} \quad & F = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}} d_{ij} \cdot f_{ij} \cdot c_{ij}, \\ \text{s.t.} \quad & \phi(i) = \left(\left(p_x^i, p_y^i\right), \left(s_x^i, s_y^i\right)\right), \left(s_x^i, s_y^i\right) \in S(i), \\ & E(i) := \left\{(x,y) \in \mathbb{R}^2 | p_x^i \leqslant x < p_x^i + s_x^i,\ p_y^i \leqslant y < p_y^i + s_y^i\right\}, \\ & d_{ij} = d\left(\left(s_x^i + \frac{p_x^i}{2}, s_y^i + \frac{p_y^i}{2}\right), \left(s_x^j + \frac{p_x^j}{2}, s_y^j + \frac{p_y^j}{2}\right)\right), \\ & E(i) \subseteq W(i) \quad \forall i \in \{1,2,\ldots,n\}, \\ & E(i) \cap E(j) = \emptyset \quad \forall i \neq j, \quad i,j \in \{1,2,\ldots,n\}, \\ & \forall i,j \in \{1,2,\ldots,n\} \exists \Delta \subseteq W \setminus \bigcup_k E(k) : E(i) \cup E(j) \cup \Delta \text{ is a connected space}, \\ & \inf \{d((x,y),(x',y')) | (x,y) \in E(i), (x',y') \notin W\} = 0 \quad \forall i \in Wall, \\ & \min \{d((x,y),(x',y')) | (x,y) \in E(i), (x',y') \in E(j)\} \geqslant d_{\min} \\ & \forall (i,j) \in BadNeighbors. \end{aligned}$$

Note that this problem formulation is not restricted to the positioning of machines within manufacturing workshops. It can be applied to other applications of FLPs, e.g. when producing printed circuit boards. In such a case the circuit board is regarded as a workshop and transistors, resistors, etc. correspond to facilities. Circuit paths can be modeled as aisles.

1.2. Literature review

Koopmans and Beckmann (1957) propose a quadratic assignment problem in which n equal sized facilities have to be placed on n equal sized locations. The objective is to minimize transportation costs between facilities. This model is often referred to be the common classical layout planning problem. It is a special case of the above-mentioned problem. Any of the n equal sized locations can be modeled as a workshop, which has the size of exactly one of the n facilities. As the quadratic assignment problem is NP-hard, our general FLP framework is NP-hard as well.

Bazaraa (1975) stated a generalized quadratic assignment problem, which incorporates facilities with unequal areas. In his model,