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Decision Support

# Electre Tri-C: A multiple criteria sorting method based on characteristic reference actions

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ABSTRACT

In this paper, a new sorting method, following a decision aiding constructive approach, is proposed. This method is called Electre Tri-C. As a sorting method, a set of categories must be defined to represent the way in which the actions that are going to be assigned to each of them should further be processed. This method is appropriate to deal with decision aiding contexts where the categories are completely ordered and each of them is defined through a single characteristic reference action. The set of characteristic actions should be co-constructed through an interactive process between the analyst and the decision maker. Electre Tri-C has been conceived to verify a set of natural structural requirements (conformity, homogeneity, monotonicity, and stability), which can be viewed as its fundamental properties. This method is composed of two joint rules, called descending rule and ascending rule, which must be used conjointly (and not separately). Each one of these rules selects only one category or a range of possible categories for a possible assignment of an action. This assignment depends on the comparison of such an action to the characteristic actions according to a chosen credibility level. Numerical examples are also presented in order to illustrate the main theoretical results provided by the method.



## 1. Introduction

Different decision problems require different approaches to solve them. We are interested in *decision aiding contexts* in which the objects of a decision (actions, alternatives,...) must be sorted, or assigned to a set of categories. Let us notice that the term "decision aiding" is used instead of "decision support", "decision making", or "decision analysis" to avoid any simplistic assimilation. The assignment of the actions to the categories is based on the evaluation of each action according to multiple criteria. In such decision aiding sorting contexts, several assumptions are appropriate to the manner that the decision aiding is considered:

**Assumption 1.** The set of categories to which the actions must be assigned to is completely ordered (from the best to the worst, from the highest priority to the lowest priority, from the most risky to the least risky, from the most consensual to the least consensual, and so on).

**Assumption 2.** Each category is defined *a priori* to receive actions, which will be or might be processed in the same way (at least in a first step).

**Assumption 3.** Each category is defined by a reference action, which is the most representative one, called characteristic reference action, or characteristic action.

Let us suppose that the *decision maker* is able, in interaction along with the *analyst*, to provide, for each criterion, the performance of each characteristic action. Let us notice that the term "decision maker" represents those in whose name or for whom the decision aiding must be given and the "analyst" represents a facilitator of the decision aiding process, which must perform her/his role in interaction with the decision maker.

For instance, the actions can be patients waiting for an Assisted Reproductive Technology (ART) treatment, credit demand files, risk zones, candidates for a job, environmental measures, or R&D projects. In case of ART treatments, the categories can be defined as the number of embryos to be transferred to the uterus of a women in order to achieve a pregnancy and to reduce the risk of multiple pregnancies at the same time. In the case of credit demand files, such actions can be accepted without additional information, accepted subject to additional information, sent to a particular department for further analysis, rejected under certain conditions, or rejected with no additional conditions at all. The method

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proposed in this paper, designated Electre Tri-C, seems appropriate to deal with this kind of situations in which the objective is not intend to discover the pre-existing categories where the studied actions would naturally be assigned to, but *to help* decision makers to assign each action to a certain category. This category should be the most appropriate taking into account its characteristics and the definition of the set of categories.

Electre Tri-C was designed to be used within the framework of a constructive approach (see Roy, 1993). It means that the decision aiding assignment model is, in a certain sense, at least co-constructed through an interactive process between the analyst and the decision maker. Firstly, this co-construction process is related to the definition of the characteristic actions, which represents the set of categories that the decision maker considers necessary to use for regrouping the actions. Secondly, this co-construction process also concerns the way of defining criteria, by assigning values to the indifference and preference thresholds, the veto thresholds (if they exist), and an intrinsic weight to each criterion in the pre-defined coherent family of criteria. Finally, as shown in the next sections, it is necessary that the analyst in agreement with the decision maker chooses a minimum credibility level to validate or not a comparison statement.

Electre Tri-C is composed of two joint rules, called descending rule and ascending rule. Each one of these rules selects only one category for a possible assignment of an action. They are used conjointly in order to highlight the highest category and the lowest category, which can appear potentially appropriate to receive an action. These two extreme categories can be the same. When they differ, this means that the assignment of such an action remains ill-determined within a range of possible categories taking into account the way that the set of characteristic actions defines the categories.

Since the late seventies, several procedures have been proposed for sorting problems (according to the Assumption 1) as the following ones: trichotomic segmentation (Moscarola and Roy, 1977), Utadis (Devaud et al., 1980; Zopounidis and Doumpos, 2002), N-Tomic (Massaglia and Ostanello, 1991), Electre Tri (Yu, 1992, 1993), filtering by preference (denoted here FPP) (Perny, 1998), multi-profile sorting by intersection sets (Norese and Viale, 2002), PairClas (Doumpos and Zopounidis, 2004), and Smaa-Tri (Tervonen et al., 2009). Let us notice that the key concept of the closeness relation, used in the FPP procedure (Perny, 1998), the Proaftn method (Belacel, 2000), and the sorting method by preference closeness (denoted here Closort) (Fernández et al., 2008), was first proposed by (Słowiński and Stefanowski, 1994) for a classification procedure using decision rules in the rough set theory framework.

The Electre Tri method can be applied to decision aiding contexts in which the Assumptions 1 and 2 are fulfilled, while replacing the Assumption 3 by the following one:

**Assumption 4.** Each category is defined by two reference actions, which represent its lower and upper bounds, called boundary reference actions, or boundary actions.

The boundary actions are introduced for modeling the frontiers between two consecutive categories. This means that the lower boundary action of a better category is also the upper boundary action of the worse consecutive category (categories are closed from below). The use of Electre Tri suppose that the decision maker is able, in interaction along with the analyst, to provide, for each criterion, the performance of each boundary action.

Electre Tri-C is, therefore, a new sorting method: the actions to be assigned are not compared to reference actions that represent lower and upper bounds of the categories, but instead they are compared to reference actions that contain the representative characteristics of each category. Each one of such categories must be defined in Electre Tri-C by a single characteristic action which defines it, instead of a pair of boundary actions like in Electre Tri. To avoid some misunderstanding, the Electre Tri method based on boundary actions will be designated henceforth by Electre Tri-B.

The rest of this paper is organized as follows. Section 2 introduces and reviews some concepts, their definitions, and notation. Section 3 is devoted to the proposed Electre Tri-C method, which contains the additional assumptions, the natural structural requirements, and the two joint rules and their foundations. Section 4 presents the properties of Electre Tri-C, including the analysis of the assignment results. Section 5 provides two numerical examples in order to illustrate the theoretical results presented in this paper. Section 6 presents a comparison with Electre Tri-B according to the basic assumptions, the related assignment rules, and the role of the reference actions, which are used to define the categories. Finally, the last section offers our concluding remarks and some avenues for future research.

## 2. Concepts, definitions, and notation

Let $\{a_1, a_2, \ldots, a_i, \ldots\}$ denote the *potential actions*. This set of actions, denoted $A$, can be completely known *a priori* or it may appear progressively during the decision aiding process. The objective is to assign these actions to a set of *completely ordered categories*, denoted $\{C_1, \ldots, C_h, \ldots, C_q\}$, with $q \geqslant 2$. Suppose that a *coherent family of n criteria*, denoted $F = \{g_1, \ldots, g_j, \ldots, g_n\}$, with $n \geqslant 3$, has been defined in order to evaluate any action considered to be assigned to a certain category (see Roy, 1996). In what follows, assume, without loss of generality, that all criteria $g_j \in F$ are to be maximized, which means that the preference increases when the criterion performance increases too.

Let us consider also that each criterion $g_j$ will be considered as a *pseudo-criterion*, which means that two thresholds are associated to $g_j$: an *indifference threshold*, $q_j$, and a *preference threshold*, $p_j$, such that $p_j \geqslant q_j \geqslant 0$. These thresholds are introduced in order to take into account the imperfect character of the data from the computation of the performances $g_j(a)$, for all $a \in A$, as well as the arbitrariness that affects the definition of the criteria. Based on the definition of such thresholds, the following binary relations can be derived for each criterion:

(i) $|g_j(a) - g_j(a')| \leqslant q_j$ represents a non-significant advantage of one of the two actions over the other, meaning that $a$ is *indifferent* to $a'$ according to $g_j$, denoted $aI_ja'$. Let $C(aIa')$ be the subset of criteria such that $aI_ja'$.

(ii) $g_j(a) - g_j(a') > p_j$ represents a significant advantage of $a$ over $a'$, meaning that $a$ is *strictly preferred* to $a'$ according to $g_j$, denoted $aP_ja'$. Let $C(aPa')$ be the subset of criteria such that $aP_ja'$.

(iii) $q_j < g_j(a) - g_j(a') \leqslant p_j$ represents an ambiguity zone. The advantage of $a$ over $a'$ is a little large to conclude about an indifference between $a$ and $a'$, but this advantage is not enough to conclude about a strict preference in favor of $a$. This means that there is an hesitation between indifference and strict preference. In such a case, $a$ is *weakly preferred* to $a'$, denoted $aQ_ja'$. Let $C(aQa')$ be the subset of criteria such that $aQ_ja'$.

Let us notice that $q_j$ can be null and/or equal to $p_j$. Furthermore, if $p_j = 0$, then any difference of performances in favor of one of the two actions over the other can be considered as significant for a strict preference on criterion $g_j$. However, this is not always true due to the imperfect character of the data and the arbitrariness that affects the definition of the criteria when using a continuous scale or even a discrete one for criterion $g_j \in F$. In the latter case,