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Decision Support

Optimal fences and joint price and inventory decisions in distinct markets with demand leakage

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ABSTRACT

This paper evaluates the simultaneous determination of price and inventory replenishment when a firm faces demand from distinct market segments. A firm utilizes fences, such as advance or nonrefundable payment, to maintain separation of its market segments; however, fences are imperfect and allow a degree of demand leakage from the higher-priced to the lower-priced market segment. We investigate the optimal structure of joint price and inventory decisions with fencing, and demonstrate that more segments is not necessarily better, especially when demand uncertainty is high in the presence of lost sales. We also show the impact of imperfect fences on the firm's profitability, and evaluate how fencing costs affect the optimal fencing decision.

1. Introduction

Revenue management has been viewed by many as one of the most important management science and operations research practices (see Bell, 1998). It emerged as an important price differentiation tool in the relatively deregulated world of the airline industry in the mid-1980s, and has since evolved into a mainstream business practice within a growing list of industries including travel, freight, media, utilities, and retail trade (see Talluri and van Ryzin, 2005). Indeed, revenue management has been credited with an incremental margin of 15–50% to the business bottom line and has contributed significant improvement in revenues (SAP White Paper, 2005).

One of the underlying principles of revenue management is to divide a single market into multiple sub-markets/segments and then set different prices in each sub-market. For example, many firms differentiate customers by leading them to different channels, such as online versus retail store, where firms set one price in the retail channel but offer discounts to online purchasers. Customers can choose to purchase the product online at a lower price with less information and longer processing times or, for a higher price, to interact with a salesperson to gain more information or expedite service. A second example occurs in the airlines industry, where airline companies sell tickets at a low price for those able to make payment in advance and are willing to accept penalties for returning those tickets, and reserves capacity for late arriving business travelers, whose lower sensitivity to price enables the airlines to set higher fares. Market segmentation, as illustrated by these examples, generally increases revenues and hence profits; however, the price difference between the market segments stirs some customers to switch segments. For example, a customer might visit a retail store to "touch and feel" a product but goes home and buys it online at a lower price.

Once a market segmentation structure has been put in place, firms use various conditions and restrictions to maintain separation of the price categories. Devices, such as less information, prolonged purchase processes, and early purchase and refund penalties, will "fence" customers into different market segments and make it difficult or time consuming for them to migrate from one market segment to another. A "fence" is a device that is designed to preserve market segmentation and limit spillover between segments; however, most fences are not perfect and allow some degree of demand "leakage" from the high-priced market segments to the low-priced segments. For instance, many customers switch to online shopping if the price is attractive enough, despite the long processing time. Some business travelers can purchase low-priced tickets by adjusting their schedules to meet the low fare restrictions. In the case of an imperfect fence, the degree of leakage is expected to increase as price difference between segments increases.

Imposing appropriate fences is crucial for the success of revenue management (Hanks et al., 2002; Kimes, 2002). However, many questions remain unanswered: How should inventory and

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prices be determined in the presence of imperfect fences? What is the impact of demand leakage on profit? Might there be optimal fences? What level of resources should a firm devote to fences?

In this study, we approach these questions in the context of a single period, stochastic inventory (or "newsvendor") framework with lost sales, where the firm sets inventory and prices when facing stochastic demand from different market segments. Instead of determining a *common* inventory level, the firm must determine a stocking quantity and a selling price for each market segment, with fences in place to enforce market segmentation. We assume the fences are not perfect and examine demand dependences between segments created by the "leakage" of customers from the high-priced market segment to the low-priced segment. Specifically, we model demand leakage across the market as a function of price difference between the markets, and investigate the impact of demand leakage on the firm's simultaneous inventory and pricing decisions. The results indicate that more customer segments do not necessarily outperform a single-segment, especially when demand uncertainty is high, in the presence of lost sales and high fencing costs. We first assume that fencing is costless and derive optimal pricing and inventory decisions. Then we introduce a couple of cost functions representing the amount the firm spends on fencing and establish the connection between fencing cost and demand leakage and determine the optimal amount that the firm should devote to fences.

The remainder of this paper is organized as follows. We review the literature in Section 2 and present the model in Section 3. The analysis of the basic model without fencing cost is performed in Section 4, followed by an extensive numerical study in Section 5. We investigate the impact of fencing cost functions on the firm's fencing decision-making in Section 6. We finally conclude in Section 7, and relegate all proofs to the appendix (see online supplement).

2. Literature review

A considerable literature has presented models where demands for different product classes occur concurrently and inventory is available to multiple demand classes. Talluri and van Ryzin (2005) presented a variety of reasons a monopolist might want to use multiple prices from the perspective of price discrimination. Gerchak et al. (1985) provided an early example of the way that revenue management principles can be applied across different businesses. They formulated a dynamic program to determine whether a bagel shop should sell a limited supply of bagels as individual items (at a low contribution) or wait for the lunch crowd and sell them as part of higher contribution combinations (for example, as part of a sandwich). Gerchak et al. noted that this problem was equivalent to the two-fare airline model, with single-arrival and constant-arrival probabilities for each type of customers. The work of Gerchak et al. (1985) was extended by Lee and Hersh (1993) to include multiple-fare situations, however, these researchers do not consider demand dependency.

A number of papers have modeled dependent demands. Pfeifer (1989) partitioned demand into two subsets (i.e., price-sensitive and price-insensitive customers), where both subsets were functions of realized demand. Brumelle et al. (1990) treated two-class dependent demands with a bivariate normal distribution. Belobaba and Weatherford (1996) introduced a probability to represent the diversion effect, which incorporated those customers who were willing to pay full fare or a restricted discount fare into a static decision rule. Sen and Zhang (1999) considered the newsboy problem with multiple demand classes, where demands were realized sequentially and demand dependency was modeled through the diversion. In particular, they analyzed a two-demand class model in which a fraction of the unsatisfied demand in one class diverted to the other, thus causing dependent sales. Talluri and van Ryzin (2004) investigated a single-leg revenue management problem in which the buyer's choice behavior was modeled explicitly. By specifying the probability of purchase for each fare product as a function of the set of fare products offered, they showed that the optimal policy was a nested allocation policy. Belobaba and Weatherford (1996) described this as diversion in its general sense. Specifically, they modeled diversion by assuming that a fixed portion of the unsatisfied lower price demand will join the higher price demand. In our models, demand dependencies are created by the "leakage" of customers from high-priced segments to low-priced segments; therefore, we investigate the impact when customers are able to circumvent the imperfect fences to achieve buy-down.

One special and well-studied case is the newsvendor problem with price effect, in which only single-period optimal decisions are considered. The newsvendor problem has been researched extensively (see Chung et al., 2008; Chung et al., 2009; Grubbström, in press; Khouja, 1999; Özer et al., 2007; Sahin et al., 2008; Sahin and Dallery, 2009; Wang et al., 2009). Whitin (1955) first examined the newsvendor problem with price effects, where selling price and stocking quantity were set simultaneously, and derived a closed-form expression for the case of uniformly distributed demand. Lau and Lau (1988) presented solution procedures for various objectives for normally distributed demand and for demand having a distribution constructed using a combination of statistical data analysis and experts' subjective estimates. Petruzzi and Dada (1999) generalized existing newsboy results for both additive and multiplicative demand cases. Our demand models extend the newsvendor problem with price effects from a single demand class to multiple demand classes, and also consider demand dependencies by modeling demand leakage across market segments as a function of price difference between segments.

A paper more related to our model is by Weatherford (1997), who considered the problem where the mean demand was assumed to be a linear function of price. Given fixed cross-elasticity in this demand function, he formulated expected contributions for up to three price classes and provided numerical results for optimal prices and inventory allocated to each price class. However, Weatherford's work focused on the tradeoff involved between computational effort and expected contribution when using heuristic decisions obtained from different assumptions. Zhang and Bell (2007) used similar demand functions to investigate the impact of demand leakage, and examined the case where demands were met from a common inventory by assuming that unmet demand was backlogged. However, they focused on the impact of demand leakage on the firm's profitability; there was no discussion about fences or how much should be devoted to the effort of reducing demand leakage. In this study, we include fences in a similar framework for inventory allocation and pricing decisions but with independent inventory and lost sales (unmet demand is lost), and go on to characterize optimal fences.

3. The model

We assume that a firm operates in a monopoly market, and its objective is to maximize its total profit by choosing selling prices and stocking quantities for all market segments. In line with Pfeifer (1989) and others, we consider the case with two-demand classes. The firm designs two market segments (e.g., two distribution channels), which have potentially different prices with fencing devices in place to prevent customers from purchasing in the lower-priced segment, if these customers should initially purchase in the high-priced segment. We use p_i to denote the retailing price in segment