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Heuristic constructive algorithms for open shop scheduling to minimize mean flow time

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Abstract

In this paper, we consider the problem of scheduling n jobs on m machines in an open shop environment so that the sum of completion times or mean flow time becomes minimal. For this strongly NP-hard problem, we develop and discuss different constructive heuristic algorithms. Extensive computational results are presented for problems with up to 50 jobs and 50 machines, respectively. The quality of the solutions is evaluated by a lower bound for the corresponding preemptive open shop problem and by an alternative estimate of mean flow time. We observe that the recommendation of an appropriate constructive algorithm strongly depends on the ratio n/m.

Keywords: Open shop scheduling; Mean flow time; Constructive heuristics

1. Introduction

In an open shop scheduling problem, a set of n jobs $J_i, i \in I = \{1, 2, \ldots, n\}$, has to be processed on a set of m machines $M_j, j \in J = \{1, 2, \ldots, m\}$. The processing of job J_i on machine M_j is denoted as operation (i, j), and the sequence in which the operations of a job are processed on the machines is immaterial. It is assumed that the processing times of all operations are given. As usual, each machine can process at most one job at a time and each job can be processed on at most one machine at a time.

Let C_i be the completion time of job $J_i, i = 1, 2, \ldots, n$. In this paper, we consider the minimization of the sum of the completion times of the jobs, also known as mean flow time minimization. Using the well-known 3-parameter classification, this problem is denoted as $O||\sum C_i$. In this and a parallel paper (see [2]), we deal with the development and comparison of heuristic algorithms for the problem under consideration. While in [2] metaheuristic algorithms (simulated annealing, tabu search and genetic algorithms) are investigated, we deal here exclusively with constructive algorithms.

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To avoid overlapping and redundancy, we refer for a detailed the literature review to the parallel paper [2]. We only mention that most papers in the literature deal with the minimization of makespan $C_{\max} = \max\{C_i | 1 \leqslant i \leqslant n\}$, and there are only a few papers dealing with mean flow time minization. Achugbue and Chin [1] proved that the corresponding two-machine open shop problem is NP-hard in the strong sense. Liaw et al. [7] considered the problem of minimizing total completion time with a given sequence of jobs on one machine (also denoted as $O|GS(1)|\sum C_i$). A branch and bound algorithm has been presented and tested on square problems with $n = m$. The algorithm was able to solve all problems with 6 jobs in 15 min on average and most problems with 7 jobs within a time limit of 50 hours with an average computation time of about 15 hours for the solved problems. This already indicates that from a practical point of view, an open shop problem with mean flow time minimization is much harder than the corresponding makespan problem, where all but one of the benchmark instances from the literature with 15 jobs and 15 machines and 7 instances with 20 jobs and 20 machines have been solved in acceptable time (see [6]).

The remainder of the paper is organized as follows. In Section 2, we introduce some basic notions (for a more detailed description of the underlying model including examples see e.g. [2] or [3]), and we derive estimates for the optimal objective function value. In Section 3 we describe several constructive algorithms for the open shop problem with mean flow time minimization. A detailed computational comparison of the algorithms is discussed in Section 4. Section 5 contains some conclusions and summarizing recommendations.

2. Basic notions and evaluation of schedules

In the following, we use the digraph $G(MO, JO)$ with operations as vertices and arcs between two immediately succeeding operations of a job or on a machine. If we place the operations of job J_i in the i-th row and the operations on machine M_j in the j-th column, then $G(MO, JO) = G(MO) \cup G(JO)$ holds where $G(MO)$ contains only horizontal arcs (describing the machine order of the jobs) and $G(JO)$ contains only vertical arcs (describing the job orders on the machines).

A combination of machine orders and job orders (MO, JO) is feasible, if $G(MO, JO)$ is acyclic. An acyclic digraph $G(MO, JO)$ is called a *sequence graph*. In this case all above graphs represent partial orders on the set of operations. Similarly as in [2,3,5,8], we describe a sequence graph $G(MO, JO)$ by its *rank matrix* $A = (a_{ij})$, i.e. the entry $a_{ij} = k$ means that a path to operation (i, j) with a maximal number of operations has k operations. Due to this property, equality $a_{ij} = k$ implies that there is no other operation with rank k in row i and column j. Moreover, the so-called *sequence property* is satisfied: 'For each $a_{ij} = k > 1$, integer $k-1$ occurs as entry in row i or column j (or both).' Now we assign the processing time t_{ij} as the weight to operation (i, j) in $G(MO, JO)$. The computation of a longest path to the vertex (i, j) with (i, j) included in an acyclic digraph $G(MO, JO)$ yields the completion time c_{ij} of operation (i, j) in the semiactive schedule $C = (c_{ij})$. We remind that a schedule is called *semiactive* if no operation can start earlier without changing the underlying sequence graph (machine and job orders).

The head r_{ij} of an operation (i,j) is defined as its earliest possible starting time according to $G(MO, JO)$, i.e. as the longest path to vertex (i, j) in this graph with vertex (i, j) not included. The advantage of the use of the rank matrix in contrast to the usual description of a solution by a permutation of the operations is the exclusion of redundancy, i.e. different rank matrices describe different solutions while different operation sequences may describe the same solution.

In [4], Bräsel and Hennes generalized the model to the preemptive case and derived a lower bound for the open shop problem $O|pmtn|\sum C_i$ which can also be taken for the nonpreemptive problem. This lower bound is as follows. Let

$$T_i = \sum_{j=1}^{m} t_{ij} \quad \text{and} \quad \overline{T}_j = \sum_{i=1}^{n} t_{ij}$$

and suppose that jobs and machines are ordered such that

$$T_1 \leqslant T_2 \leqslant \ldots \leqslant T_n \quad \text{and} \quad \overline{T}_1 \leqslant \overline{T}_2 \leqslant \ldots \leqslant \overline{T}_m.$$