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On path correlation and PERT bias

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Abstract

Most studies of project time estimation assume that (a) activity times are mutually independent random variables; many also assume that (b) path completion times are mutually independent. In this paper, we subject the impact of both these assumptions to close scrutiny. Using tools from multivariate analysis, we make a theoretical study of the direction of the error in the classical PERT method of estimating mean project completion time when correlation is ignored. We also investigate the effect of activity dependence on the normality of path length via simulation.

Keywords: Project management; PERT; Correlation; Project networks; Slepian's inequality

1. Introduction

The critical path method of estimating project completion time and its probabilistic version, PERT, are widely used by project management practitioners. It is important for practitioners to have sound *a priori* estimates of project completion time because cost planning and resource allocation decisions hinge crucially on these estimates.

Ever since the 1950s, when the PERT method was formulated, researchers have attempted to construct a rigorous theoretical foundation for it. It is now well accepted that PERT gives useful estimates. However, there is still work to be done on isolating potential sources of bias in its application and on developing guidelines linking specific assumptions to their impact on PERT bias. The present paper studies correlation between activities and between paths as sources of PERT bias. For comprehensive surveys of research on project time estimation, see Elmaghraby [3] and Slowinski and Weglarz [11], among many other sources.

Most studies of project time estimation assume that activity times are mutually independent random variables; many also assume that path completion times are mutually independent. In this note, we study PERT bias in projects with correlated activities and paths using tools from multivariate analysis. We also report on a simulation experiment that sheds some light on the effect of activity dependence on the normality of path length distribution.

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2. The impact of correlation on PERT bias

Recall that the PERT method estimates the mean project completion time to be $\text{Max}(\text{E}L_1, \ldots, \text{E}L_N)$, where L_i is the length of path i in the project network; this is always smaller than E Max $(L_1, \ldots, L_N)$, the true mean project completion time. The difference between these two metrics is the magnitude of PERT bias. The PERT method therefore always underestimates true mean project duration, a fact that is widely known. Colloquially speaking, PERT yields optimistic estimates. MacCrimmon and Ryavec [7] (hereafter referred to as M&R) inferred from numerical examples that *the higher the correlation in a network because of overlapping paths, the smaller the magnitude of PERT bias*, but this conclusion has not, to our knowledge, been subjected to rigorous scrutiny. We show that whether or not this rule of thumb is true depends on the nature of the correlation between activities and paths, and on the distribution of path lengths and activity durations.

M&R used the following example to illustrate their point:

In Fig. 1, the path lengths in Project A are the following:

Path 1: $x_1 + x_2$
Path 2: x_3
Path 3: $x_4 + x_5$

Note that the path lengths are independent random variables if all the activities are mutually independent. The path lengths in Project B are as follows:

Path 1: $x_1 + x_2$
Path 2: $x_1 + y + x_5$
Path 3: $x_4 + x_5$

Note that although the projects have two path lengths that are identical, the two projects differ in an essential way: the path lengths in Project B are pairwise dependent because of the connecting activity with duration y. M&R conjecture that the correlated paths in Project B should buy it a smaller value of PERT bias compared with Project A provided the mean lengths of Path 2 are the same in both projects. They verify their conjecture with specific discrete distributions for the activity times. However, the following counterexample shows that even for the specific networks constructed by M&R, a correlated project may carry a higher PERT bias than an independent project.

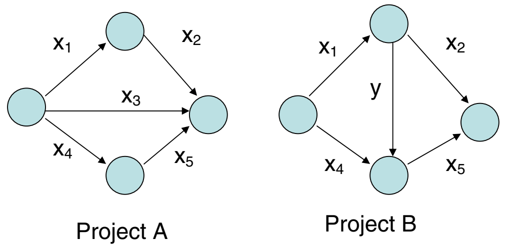

Fig. 1. Project networks for counterexample.

We fix the activity times to the following values: $x_1 = 2$ with probability 1/2 and 4 with probability 1/2 , $x_2 = 6$, $x_3 = 9$, $x_4 = 4$, $x_5 = 5$, $y = 0$ with probability 1/2 and 2 with probability 1/2 . Note that Path 1 and Path 3 in Project A are identical to the corresponding paths in Project B; Path 2 in Project A has the same mean completion time as Path 2 in Project B. A simple computation shows that the completion time of Project A is 9 with probability 1/2 and 10 with probability 1/2, giving a mean completion time of 9.5. On the other hand, the completion time of Project B is 9 with probability 1/2, 10 with probability 1/4 and 11 with probability 1/4 giving a mean completion time of 9.75. This example shows that correlation between paths may inflate PERT bias.

In the remainder of the paper, we attempt to find broad patterns between PERT bias and correlation. We deal separately with two categories of projects: (a) projects with independent activities giving rise to correlated paths, and (b) projects in which the activities are correlated *per se.*

2.1. Projects with independent activities

We begin with a result on PERT bias in networks with independent activities and overlapping paths (that is, paths with at least one activity in common). Although we have not seen it explicitly stated in the literature, it may be easily inferred from the theoretical framework developed in [2]. The proof is a direct application of some properties of associated random vectors. A random vector $\mathbf{X} = (X_1, \ldots, X_N)$ is said to be *associated* if $\text{Cov}(f(\mathbf{X}), g(\mathbf{X})) \geqslant 0$ for all coordinate-wise increasing functions $f(\cdot)$ and $g(\cdot)$ from R^N to R. If $\mathbf{X}$ is associated, it follows from the definition that the components of $\mathbf{X}$ are pairwise positively correlated. The other facts about associated random vectors used in the proof of the following theorem are taken from Esary, Proschan and Walkup [4].