European Journal of Operational Research 204 (2010) 218–228

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Discrete Optimization

A new branch-and-price algorithm for the traveling tournament problem

Stefan Irnich *

Chair of Logistics Management, Johannes Gutenberg University Mainz, Jakob-Welder-Weg 9, D-55128 Mainz, Germany

ARTICLE INFO

Article history:
Received 29 April 2009
Accepted 27 October 2009
Available online 1 November 2009

Keywords:
Timetabling
Sports league scheduling
Traveling tournament problem
Column generation
Branch-and-price

ABSTRACT

The traveling tournament problem (TTP) consists of finding a distance-minimal double round-robin tournament where the number of consecutive breaks is bounded. For solving the problem exactly, we propose a new branch-and-price approach. The starting point is a new compact formulation for the TTP. The corresponding extensive formulation resulting from a Dantzig-Wolfe decomposition is identical to one given by Easton, K., Nemhauser, G., Trick, M., 2003. Solving the traveling tournament problem: a combined interger programming and constraint programming approach. In: Burke, E., De Causmaecker, P. (Eds.), Practice and Theory of Automated Timetabling IV, Volume 2740 of Lecture Notes in Computer Science, Springer Verlag Berlin/Heidelberg, pp. 100–109, who suggest to solve the tour-generation subproblem by constraint programming. In contrast to their approach, our method explicitly utilizes the network structure of the compact formulation: First, the column-generation subproblem is a shortest-path problem with additional resource and task-elementarity constraints. We show that this problem can be reformulated as an ordinary shortest-path problem over an expanded network and, thus, be solved much faster. An exact variable elimination procedure then allows the reduction of the expanded networks while still guaranteeing optimality. Second, the compact formulation gives rise to supplemental branching rules, which are needed, since existing rules do not ensure integrality in all cases. Third, non-repeater constraints are added dynamically to the master problem only when violated. The result is a fast exact algorithm, which improves many lower bounds of knowingly hard TTP instances from the literature. For some instances, solutions are proven optimal for the first time.

1. Introduction

The traveling tournament problem is the problem of finding a double round-robin schedule that minimizes the overall distance traveled by all teams such that, for each team, the number of consecutive home stands and consecutive away games is bounded. The TTP was introduced by Easton et al. (2001) as an artificial sports league scheduling problem. Since then, it has attracted numerous researchers, probably because of its fast growing difficulty.

Formally, an even number $n \in 2\mathbb{N}$ of teams is given. Let $T := \{1, 2, \dots, n\}$ denote the set of teams. In a single round-robin tournament, each team t plays against each of its opponent teams $T_{-t} := T \setminus \{t\}$ once. Assuming that the tournament takes place on a minimum number of matchdays (in the following called "time slots"), there are $n/2$ games in each of the $\bar{n} := n - 1$ time slots. In a double round-robin tournament, each team plays against each other team twice, once at home and once away. Consequently, there are $2\bar{n}$ time slots with again $n/2$ games in each slot. In the following, the time slots $S = \{1, 2, \dots, 2\bar{n}\}$ are indexed by s.

For each team, the sequence of consecutive games played (home or at an opponent's venue) implies a tour: We identify teams and their venues and use indices $i, j \in T$ to refer to venues. A tour $p = (i_1, i_2, \dots, i_{2\bar{n}}) = (i_s)_{s \in S}$ of team $t \in T$ contains each opponent venue $i \in T_{-t}$ exactly once (away games) and the home venue $i = t$ exactly $\bar{n}$ times (home games). A *break* occurs if a home game is followed by another home game or if an away game is followed by another away game, i.e., $i_s = i_{s+1} = t$ or $i_s, i_{s+1} \in T_{-t}$ for a time slot $s < 2\bar{n}$. In the TTP, the number of consecutive home stands and consecutive away games is bounded by L and U, i.e., the number of consecutive breaks is bounded by $L - 1$ and $U - 1$. Since all instances from the literature have $L = 1$, we solely focus on the upper bound U. Moreover, there are (optional) no-repeater constraints (NRCs) stating that the game t against t' must not be followed by the return game t' against t for any pair of teams $t, t' \in T$.

The objective of the TTP is distance minimization over all teams. Distances $D = (d_{ij})_{i,j \in T}$ between the venues are assumed symmetric and non-negative. Because each team t initially starts at home ($i_0 = t$) and finally returns home ($i_{2\bar{n}+1} = t$), the distance traveled along a tour $p = (i_1, i_2, \dots, i_{2\bar{n}})$ is $\sum_{s=0}^{2\bar{n}} d_{i_s, i_{s+1}}$. Summing up, an instance of TTP is defined by distances $D = (d_{ij})$, an integer U, and optional NRCs. The task is to compute a distance-minimal set of

* Tel.: +49 61313922007; fax: +49 61313922097.
E-mail address: irnich@uni-mainz.de

0377-2217/$ - see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2009.10.024

break-feasible tours that compose a double round-robin tournament (without repeater games).

The recent survey by Rasmussen and Trick (2008) devotes a full section to the TTP and gives a comprehensive overview of state-of-the-art approaches. While there is a large variety of metaheuristics available (see survey), the literature on exact algorithms is scarce: Based on the so-called independent lower bound (ILB) relaxation, Easton et al. (2001) are able to solve TTP instances with $n = 4$ and $n = 6$ teams. The same authors present in (Easton et al., 2003) a column-generation approach for TTP without NRCs, where the subproblems, one for each team, consist of generating least-cost (=reduced cost) tours. Subproblems are solved by constraint programming (CP), and integrality of the overall solution is enforced by branch-and-bound. A method to improve lower bounds is presented by Urrutia et al. (2007), but (to the best of our knowledge) no exact approach has been implemented based on this idea. The special case of mirrored TTP with uniform distances is treated in (Urrutia and Ribeiro, 2004), where instances with up to $n = 12$ teams are solved to optimality. Cheung (2008) is able to solve small ($n \leqslant 8$) mirrored TTP benchmark problems from http://mat.tepper.cmu.edu/TOURN (with non-constant distances) using a two-phase approach: First, all different one-factorizations are computed and then, for each of them, a timetable-constrained distance minimization problem is solved afterwards. Cheung (2009) proposes a Benders decomposition approach, which improves the lower bound for larger instances of the same type, i.e., for n between 10 and 24. It seems rather unlikely that these methods are successfully applicable to general or larger TTP instances. This motivated our research on fast exact approaches for the TTP.

This paper is structured as follows: Section 2 presents the new compact formulation for the TTP. The proposed Dantzig-Wolfe decomposition of this model and the resulting master and pricing problems are derived in Section 3. Section 4 devises the corresponding solution methods for the integer programming master. Computational results are discussed in Section 5 and final conclusions are drawn in Section 6.

2. Compact formulation

In integer column generation, a well-structured compact (=original) formulation is extremely important for devising branching rules and adding valid inequalities to the extensive (=column-generation) formulation (cf. Lübbecke and Desrosiers, 2005; Spoorendonk, 2008). The basic idea of the new compact formulation for the TTP is to represent the movement of each team, from venue to venue, by a path in a time-discrete network. Fig. 1 depicts the network for a TTP with $n = 4$ teams. For each time slot $s \in S$, a given team $t \in T$ visits one of the venues $i \in T$, i.e., its own home venue or any opponent's venue. The nodes of the network of team t are therefore

$$V^t = \{v_{is} : i \in T, s \in S\} \cup \{v_{t0}, v_{t,2\bar{n}+1}\}.$$

The two extra nodes v_{t0} and $v_{t,2\bar{n}+1}$ are source and sink. They model the fact that team t always starts at home and returns home at the end. The possible movements in space and time are given by arcs $(v_{is}, v_{j,s+1})$ with the meaning that team t is traveling from venue i at time s to venue j, where the next game takes places at time $s+1$. To lighten the notation, arcs $(v_{is}, v_{j,s+1})$ are encoded by triplets (i,j,s), where the set of all feasible triplets for team t is

$$\begin{aligned} A^t = &\{(t,j,0) : j \in T\} \cup \{(i,t,2\bar{n}) : i \in T\} \\ &\cup \{(i,j,s) : i,j \in T, (i \neq j \text{ or } i = j = t), s \in S \setminus \{2\bar{n}\}\} \end{aligned}$$

The subset $B^t = \{(i,j,s) \in A^t : (i = j = t, s \neq 0, 2\bar{n}) \text{ or } i,j \in T_{-t}\} \subset A^t$ represents home stands and consecutive away games and, thus, defines the set of *break arcs*. Note that a home game in $s = 1$ or $s = 2\bar{n}$ does *not* impose a break. We use A^t to refer to arcs of the network $\mathcal{N}^t = (V^t, A^t)$ and also to index the corresponding decision variables $x^t_{ijs} \in \{0,1\}$ of the following compact formulation:

$$z_{ttp} = \min \sum_{t \in T} \sum_{(i,j,s) \in A^t} d_{ij} x^t_{ijs} \tag{1}$$

$$\text{s.t.} \quad \sum_{i:(i,j,s-1) \in A^t} x^t_{ij,s-1} - \sum_{i:(j,i,s) \in A^t} x^t_{jis} = 0 \quad \text{for all } t,j \in T, s \in S \tag{2}$$

$$\sum_{s \in S} \sum_{j:(i,j,s) \in A^t} x^t_{ijs} = 1 \quad \text{for all } t \in T, i \in T_{-t} \tag{3}$$

$$\sum_{u=0}^{U-1} \sum_{(i,j,s+u) \in B^t} x^t_{ij,s+u} \leqslant U - 1 \quad \text{for all } t \in T, s \in S : s \leqslant 2\bar{n} - U \tag{4}$$

$$\sum_{i \in T_{-t}} \sum_{j:(i,j,s) \in A^t} x^t_{ijs} + \sum_{t' \in T_{-t}} \sum_{j:(t,j,s) \in A^{t'}} x^{t'}_{tjs} = 1 \quad \text{for all } t \in T, s \in S \tag{5}$$

$$x^t_{ijs} \in \{0,1\} \quad \text{for all } t \in T, (i,j,s) \in A^t \tag{6}$$

The objective (1) is the minimization of the overall distance traveled by all teams. Flow conservation for each team is implied by (2), constraints (3) state that all teams must visit all opponent venues exactly once, and constraints (4) limit the number of consecutive breaks. The coupling constraints (5) are the crucial part of the model: They guarantee that each team t plays a game in each time slot s, either playing away against an opponent t' (first sum) or playing home as the opponent of another team t' (second sum).

One advantage of this formulation is that NRCs are simple to add: $x^t_{tt's} + x^{t'}_{tt's} \leqslant 1$ must hold for all $t \in T, t' \in T_{-t}, s \in S, s < 2\bar{n}$. It means that teams t and t' are not allowed to play against each other in consecutive time slots, first in slot s home at t, directly followed by the return game in slot $s+1$ home at t'. By swapping the role of t and t' and anticipating that the four corresponding arcs are pairwise incompatible, NRCs can be lifted to

$$x^t_{tt's} + x^{t'}_{tt's} + x^{t'}_{t'ts} + x^t_{t'ts} \leqslant 1 \quad \text{for all } t,t' \in T, t < t'; \quad s \in S, s \neq 2\bar{n}. \tag{7}$$

These are $n\bar{n}(2\bar{n}-1)/2 = \mathcal{O}(n^3)$ lifted NRCs. Thus, (1)–(7) is the compact formulation for the TTP with NRCs.

3. Extensive formulation

The extensive formulation consists of two parts: First, we briefly state the master program, which is identical to one used in Easton et al. (2003). The new aspect is the incorporation of the NRCs, which are directly derived from the compact formulation presented above. Second, we discuss the structure of the subproblems.

3.1. Master problem

The application of the Dantzig–Wolfe decomposition principle to the model (1)–(7) is straightforward: Note that the only constraints involving more than one team are the coupling constraints (5) and the NRCs (7). Therefore, constraints 2, 3, 4 and (6) define the domains of the subproblems. They decompose into n domains and corresponding subproblems, one for each team $t \in T$: Let $P^t = \{(x^t_{ijs}, (i,j,s) \in A^t$: satisfying 2, 3, 4 and (6)$\}$. The set P^t is the set of feasible paths from source v_{t0} to sink $v_{t,2\bar{n}+1}$ in the network $\mathcal{N}^t$. Such a path must be break-feasible and visit each opponent venue exactly once. The cost of a tour $p = \left(\bar{x}^t_{ijs}\right) \in P^t$ is $c_p = \sum_{(i,j,s) \in A^t} d_{ij} \bar{x}^t_{ijs}$.

Easton et al. (2003) were the first to present a column-generation formulation based on the tour variables $\lambda^t_p, p \in P^t$ for the TTP, but without deriving it from an original compact formulation. We define $P^t_{t's}$ to be the subset of tours in P^t, where team t plays away in slot s against team t', i.e., $p \in P^t_{t's}$ visits venue of t' in slot s (p touches the node $v_{t's}$). The extensive formulation is as follows: