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Discrete Optimization

Discretized formulations for capacitated location problems with modular distribution costs

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ABSTRACT

In this paper we study a discretization reformulation technique in the context of a facility location problem with modular link costs. We present a so-called 'traditional' model and a straightforward discretized model with a general objective function whose variable coefficients are computed by solving a simple knapsack problem. We show that the linear programming relaxation of the discretized model dominates the linear programming relaxation of the original model. The discretized model suggests quite intuitive valid inequalities that considerably improve the linear programming relaxation of the original model. Computational results based on randomly generated data show that in the context of problems with modular costs, the proposed discretized models perform significantly better than the 'traditional' models. An important outcome of our research is the result, whose proof is also presented in this paper, that a restricted version of the discretized model gives an extended description of the convex hull of the integer solutions of a subproblem that usually arises in network design problems with modular costs.

1. Introduction

In this paper we study a capacitated facility location problem that can be described as follows. Let I denote the set of potential locations, each with capacity Q, and let J be the set of customers or demand points. For each demand point $j \in J$ let d_j designate the corresponding (integer) demand. We also let f_i denote the fixed cost for establishing a facility in location i. We assume that several modules, of several sizes, can be installed on each facility-customer link. We denote by $\{1, \dots, L\}$ the set of different types of modules. Let C^l designate the capacity of module type l and g^l the corresponding cost ($l \in \{1, \dots, L\}$). Finally, let c_{ij} denote the unitary distribution cost on the link (i, j). We want to guarantee that all demand is satisfied with minimum cost (the cost of a solution is the cost of establishing the facilities and the fixed and variable costs of the links between customers and facilities).

The problem studied in this paper arises in a facility location context and is motivated by the fact that in many real applications, the costs associated with the links between facilities and customers are much more complex than the straightforward linear term that is often considered in the literature. The importance of considering more general distribution costs arises, for instance, when different means of transportation are available for delivering goods from facilities to customers. This also implies that a facility-customer link may have to be set with a combination of different capacities/modules. This is the case when, for example, we have different types of trucks that are available to supply the customers. If more than one truck of each size is available, we obtain a situation with capacitated modular links. Another example can be found in telecommunications when different cables are available to build the link between one demand point and a concentrator.

Several papers can be found in the literature in which models with general costs associated with the facilities are presented. For instance, Correia and Captivo (2003) study the problem in which the capacity for each facility must be chosen from a set of available capacities, each of which is associated with a different cost. Hajiaghayi et al. (2003) consider a set-up cost associated with the facilities that is a non-decreasing function of the number of demand points served by the facility. Wu et al. (2006) consider a general set-up cost that is function of the size of the facility. Gouveia and Saldanha-da-Gama (2006) propose formulations for unit-demand modular concentrator location problems.

On the other hand, the literature in locational analysis is rather scarce for location models involving more general structures for the distribution costs. In fact, we are only aware of the paper by Holmberg (1999) that considers convex distribution/assignment costs for an uncapacitated location problem and the paper by Melkote and Daskin (2001) that considers a model including set-up shipment costs for the links as well as a linear cost depending

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on the quantity to be shipped. To the best of our knowledge, the location problem studied in this paper has never been considered in the literature.

To model the problem, we use models with discretized variables as in Gouveia (1995) and Gouveia and Saldanha-da-Gama (2006). There are two reasons for using such models: (i) as noted in Gouveia and Saldanha-da-Gama (2006), modular costs are easily associated with the discretized variables and (ii) the discretized variables suggest valid inequalities which are quite intuitive with the new set of variables and which help to considerably improve the LP relaxation of the original non-discretized model. In fact, we will show that the use of these valid inequalities has contributed to solving instances that could not be solved with the original model.

As a side-effect of our research, we also show that a restricted version of the discretized model gives an extended description of the convex hull of the integer solutions of a subproblem that usually arises in network design problems with modular costs (see, for instance, Magnanti et al. (1995) and the references in Frangioni and Gendron (2009)). This convex hull is easy to describe when the problem uses two variables for each element i (one "modular" variable and one integer variable) but becomes quite elusive when more than one modular variable is needed for each element i.

The remainder of this paper is organized as follows. We start by formulating the capacitated facility location problem with modular distribution costs. In Section 3 we present a discretized formulation for the problem. A comparison between the formulations is presented in Section 4. In Section 5 we present and discuss valid inequalities for both the 'classical' and the discretized models. In Section 6 we report the results of computational tests based on randomly generated data. We conclude the paper with some conclusions about the work developed.

For any model P we denote by $v(P)$ its optimal value and by $v(\overline{P})$ the optimal value of its linear programming relaxation.

2. The capacitated location problem with modular distribution costs

In order to formulate the capacitated location problem with modular distribution costs, we consider the following decision variables: (i) integer variables x_{ij} indicating the quantity that facility i sends to demand point $j (i \in I, j \in J)$, (ii) variables u^l_{ij} which indicate the number of modules of type l to be installed in link (i,j) and (iii) binary variables y_i indicating whether a facility is established at i $(i \in I)$. The problem can be formulated as follows:

$$(\text{M})\text{Min} \quad \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J}\sum_{l=1}^{L} g^l u^l_{ij} + \sum_{i\in I}\sum_{j\in J} c_{ij}x_{ij} \tag{2.1}$$

$$\text{s.t.} \quad \sum_{i\in I} x_{ij} = d_j \quad j \in J \tag{2.2}$$

$$\sum_{j\in J} x_{ij} \leqslant Q y_i \quad i \in I \tag{2.3}$$

$$x_{ij} \leqslant \sum_{l=1}^{L} C^l u^l_{ij} \quad i \in I, \quad j \in J \tag{2.4}$$

$$u^l_{ij} \geqslant 0 \text{ and integer} \quad i \in I, \quad j \in J, \quad l \in \{1,\ldots,L\} \tag{2.5}$$

$$x_{ij} \geqslant 0 \text{ and integer} \quad i \in I, \quad j \in J \tag{2.6}$$

$$0 \leqslant y_i \leqslant 1 \text{ and integer} \quad i \in I \tag{2.7}$$

The objective function (2.1) minimizes the total cost, which includes the cost for establishing the facilities and the cost associated with the links. Constraints (2.2) guarantee that all the demand is satisfied while constraints (2.3) are the facility capacity constraints. Constraints (2.4) state that the modules installed on link (i,j) have enough combined capacity to support the traffic on that link. Constraints (2.5), (2.6) and (2.7) are domain constraints. Several classes of valid inequalities to improve the linear programming relaxation of model (M) are known. Some of these inequalities will be described in Section 5.

3. The discretized model

A reformulation of (M) can be obtained by using the set of discretized variables $z^1_{ij},\ldots,z^Q_{ij}$ for each link (i,j) (see, for instance, Gouveia (1995)). $z^q_{ij} (q = 1,\ldots,Q)$ indicates whether q units are shipped through link (i,j). The "discretized" formulation is as follows

$$(\text{DM+})\text{Min} \quad \sum_{i\in I} f_i y_i + \sum_{i\in I}\sum_{j\in J}\sum_{q=1}^{Q} \left(\alpha^q_{ij} + qc_{ij}\right) z^q_{ij} \tag{3.1}$$

$$\text{s.t.} \quad \sum_{i\in I}\sum_{q=1}^{Q} q z^q_{ij} = d_j \quad j \in J \tag{3.2}$$

$$\sum_{j\in J}\sum_{q=1}^{Q} q z^q_{ij} \leqslant Q y_i \quad i \in I \tag{3.3}$$

$$\sum_{q=1}^{Q} z^q_{ij} \leqslant 1 \quad i \in I, \quad j \in J \tag{3.4}$$

$$0 \leqslant z^q_{ij} \leqslant 1 \text{ and integer} \quad i \in I, \quad j \in J, \quad q = 1,\ldots,Q \tag{3.5}$$

$$0 \leqslant y_i \leqslant 1 \text{ and integer} \quad i \in I \tag{3.6}$$

where α^q_{ij} denotes the lowest set-up cost for link (i,j) conditional to an installed capacity of at least q units. As noted in Gouveia and Saldanha-da-Gama (2006), for a problem with modular costs, the coefficients of the discretized variables in the objective function can be obtained by solving an associated knapsack problem. For the case of the problem under study, α^q_{ij} corresponds to the optimal value of the following knapsack problem

$$(Kp^q_{ij})\text{Min} \quad \sum_{l=1}^{L} g^l \mu^l_{ij}(q) \tag{3.7}$$

$$\text{s.t.} \quad \sum_{l=1}^{L} C^l \mu^l_{ij}(q) \geqslant q \tag{3.8}$$

$$\mu^l_{ij} \geqslant 0 \text{ and integer} \quad l \in \{1,\ldots,L\} \tag{3.9}$$

We denote by $\mu^{1*}_{ij}(q), \mu^{2*}_{ij}(q),\ldots,\mu^{L*}_{ij}(q)$ the optimal solution values of the variables of (Kp^q_{ij}). In formulation (DM+), constraints (3.2) and (3.3) correspond to constraints (2.2) and (2.3). The consistency constraints (3.4) guarantee that in each link (i,j) at most one variable z^q_{ij} is equal to one. The model (DM+) illustrates two key ideas for using discretization namely, (i) the fact that the discretized variables allow modeling easily non-standard objective functions and (ii) that the discretized variables (or more precisely, the fact that the discretized variables combine together information given by two different variables in the original model) make unnecessary coupling constraints such as constraints (2.4) of the conventional model (M).

4. Comparing the linear programming relaxation of the models

The fact that we have now two models for the proposed modular version of the capacitated location problem, the original model (M) and the discretized model (DM+), raises the following questions:

Q1 What is the relationship between the linear programming relaxation of the models (M) and (DM+)?