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Continuous Optimization

Convex fuzzy mapping with differentiability and its application in fuzzy optimization

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Abstract

The concepts of differentiability, convexity, generalized convexity and minimization of a fuzzy mapping are known in the literature. The purpose of this present paper is to extend and generalize these concepts to fuzzy mappings of several variables using Buckley–Feuring approach for fuzzy differentiation and derive Karush–Kuhn–Tucker condition for the constrained fuzzy minimization problem.

Keywords: Fuzzy numbers; Differentiation; Convex analysis; KKT condition

1. Introduction

The theory of fuzzy differential calculus has been discussed by many researchers like Goetschel–Voxman [8], Seikkala [12], Puri–Ralescu [11], Dubois–Prade [5,6], and Friedman–Ming–Kandel [7]. A comparison of these various definitions has been discussed by Buckley–Feuring [3]. Goetschel–Voxman, Puri–Ralescu, and Friedman–Ming–Kandel have used non-standard fuzzy subtraction to define derivative of a fuzzy mapping. Buckley–Feuring [2,3] also have defined the derivative of a fuzzy mapping using left and right-hand functions of its α-level sets and established sufficient conditions for the existence of fuzzy derivative. Existence of Buckley–Feuring derivative implies the existence of above derivatives. Hence in this paper we accept the concept of differentiability of fuzzy mapping due to Buckley–Feuring [2,3].

Nanda and Kar [10] introduced the concept convexity for fuzzy mapping and proved that a fuzzy mapping is convex if and only if its epigraph is a convex set. Yan–Xu [16] have discussed convexity, quasiconvexity of fuzzy mappings by considering the concept of ordering due to Goetschel–Voxman [8]. Syau in [13] has proved

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some results on convex fuzzy mapping and in [14], introduced the concept of differentiability, generalized convexity such as pseudoconvexity and invexity for fuzzy mappings of several variables. His approach is parallel to Goetschel–Voxman approach for fuzzy mapping of single variable in which the set of fuzzy numbers are embedded in a topological vector space.

Nanda–Kar [10] have defined the concept of quasiconvex fuzzy mapping and have discussed some applications to optimization. However, since the set of fuzzy numbers is a partially ordered set, it might happen that two fuzzy numbers may not be comparable (see Definition 3.10). Thus, in such case one is not sure what is the maximum or minimum of two fuzzy numbers (that is, when they are not comparable). So to overcome this difficulty, Syau [13] has taken a different approach and defined the supremum and infimum for a pair of fuzzy numbers (see Lemma 2.6). Accordingly he modified the definition of a quasiconvex fuzzy mapping of Nanda–Kar [10]. However, when we deal with the problem of minimization of a fuzzy mapping by considering the infimum defined by Syau [13], we might get a unique infimum of the fuzzy mapping (provided it is bounded). But in the process we might loose the solution, or in other words, there might not be any point (solution) (in the domain) at which the fuzzy mapping will have the value equal to infimum (see Example 3.10). So to overcome this difficulty we have modified the definition of quasiconvex fuzzy mapping in Section 4, which is different from Syau [13] as well as Nanda and Kar [10].

Section 3 deals with the minimization of a fuzzy mapping and also we introduce the gradient of a fuzzy function, directional derivative of a fuzzy function and establish the condition for a local minimum of a fuzzy differentiable function. The concept of convex fuzzy mapping, generalized convex fuzzy mapping such as quasiconvexity, strict quasiconvexity, strong quasiconvexity, pseudoconvexity using differentiability concept is introduced in Section 4. Section 5 deals with the Karush–Kuhn–Tucker optimality conditions for the constrained fuzzy minimization problem.

2. Preliminaries

We first quote some preliminary notations, definitions and results which will be needed in the sequel.

Definition 2.1. Let $\mathbb{R}$ denote the set of all real numbers. A fuzzy number is a mapping $\tilde{u} : \mathbb{R} \to [0, 1]$ with the following properties:

1. $\tilde{u}$ is normal, that is, the core of $\tilde{u} = \text{core}(\tilde{u}) = \{x \in \mathbb{R} : \tilde{u}(x) = 1\}$ is not empty,
2. $\tilde{u}$ is upper semi-continuous,
3. $\tilde{u}$ is convex, that is,
$$\tilde{u}(\lambda x + (1 - \lambda y) \geqslant \min\{\tilde{u}(x), \tilde{u}(y)\}$$
for all $x, y \in \mathbb{R}, \lambda \in [0, 1]$,
4. the support of $\tilde{u}$, supp $\tilde{u} = \{x \in \mathbb{R} : \tilde{u}(x) > 0\}$ and its closure $cl(\text{supp }\tilde{u})$ is compact.

Let $\mathscr{F}$ be the set of all fuzzy numbers on $\mathbb{R}$. The α-level set of a fuzzy number $\tilde{u} \in \mathscr{F}$, $0 \leqslant \alpha \leqslant 1$, denoted by $\tilde{u}[\alpha]$, is defined as

$$\tilde{u}[\alpha] = \begin{cases} \{x \in \mathbb{R} : \tilde{u}(x) \geqslant \alpha\} & \text{if } 0 < \alpha \leqslant 1, \\ cl(\text{supp}\,\tilde{u}) & \text{if } \alpha = 0. \end{cases}$$

It is clear that the α-level set of a fuzzy number is a closed and bounded interval $[u_\star(\alpha), u^\star(\alpha)]$, where $u_*(\alpha)$ denotes the left-hand end point of $\tilde{u}[\alpha]$ and $u^*(\alpha)$ denotes the right-hand endpoint of $\tilde{u}[\alpha]$.

Also any $m \in \mathbb{R}$ can be regarded as a fuzzy number $\tilde{m}$ defined by

$$\tilde{m}(t) = \begin{cases} 1 & \text{if } t = m, \\ 0 & \text{if } t \neq m. \end{cases}$$

In particular, the fuzzy number $\tilde{0}$ is defined as $\tilde{0}(t) = 1$ if $t = 0$, and $\tilde{0}(t) = 0$ otherwise.

Thus a fuzzy number $\tilde{u}$ can be identified by a parameterized triples

$$\{(u_*(\alpha), u^*(\alpha), \alpha) : 0 \leqslant \alpha \leqslant 1\}.$$