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Discrete Optimization

Cyclic multiple-robot scheduling with time-window constraints using a critical path approach

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Abstract

An automated production system is considered in which several robots are used for transporting parts between workstations following a given route in a carousel mode. The problem is to maximize the throughput rate. Extending previous works treating scheduling problems for a single robot, we consider a more realistic case in which workstations are served by multiple robots. A graph model of the production process is developed, making it possible to apply PERT–CPM solution techniques. The problem is proved to be solvable in polynomial time.

Keywords: Production–transportation process; Cyclic scheduling; Robotic scheduling; Parametric PERT–CPM model; Time windows; Critical path

1. Introduction

Industrial robots offer an effective means of utilizing high technology to make manufacturing cells more profitable and flexible. Today, they are widely used in industrial machinery, automobile, aircraft and microelectronics industries. However, planning, scheduling and synchronization of operations in such systems

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are extremely sophisticated. Proficient computer-aided decision making and efficient computer-based algorithms are needed to take proper account of such a degree of sophistication. A competent schedule, synchronizing the activities of machines and robots, may increase the throughput rate, reduce work-in-process, and reduce production costs [1,3–5,17].

A practical problem that motivated this study is encountered in an automated electro-chemical processing line *Degem Mechatronics-2000* running in the Mechatronics Lab of Holon Academic Institute of Technology. It performs coating processes on aluminium and includes a series of several baths used for cleaning, acid activating, washing and coating the workpieces. Computer-controlled robots ("hoists") are used to transport workpieces through the baths and for moving the workpieces in and out of the process. In real-life instances, the number of baths may vary from 6 to 50; processing times vary from one bath to another in the range from 15 seconds to 5 minutes whereas delivering/loading/unloading operations vary from 10 to 30 seconds. In this system, all the robots as well as all the workpieces are identical, and all robots are moving one after another in a carousel mode, which will be explained in more detail in the next section. Each robot performs a fixed sequence of moves repeatedly. A repeated sequence of moves is called a *robot cycle*. Cyclic scheduling deals with sets of tasks that have to be performed infinitely often. When lot sizes for workpieces are very large (as it is the case in a mass production in the electrochemical industry, where the lot size may be counted in hundreds and even thousands of identical items), the entire production process can be modelled as an 'infinite' cyclic process. A common objective is to minimize the cycle time, which, in turn, maximizes the throughput. The problem, in its general form, is NP-hard [15,12]. Extensive surveys of this problem can be found in [19,4,3].

For many decades, attention of researchers has been attracted by a special case of the robotic scheduling problem in which one robot is used for serving a line, the robot's route is known, and durations of technological operations lie in prescribed intervals. The first linear programming (LP) model has been suggested, to the best of our knowledge, in the 1970s by Livshitz et al. [15]. Lei [11] derives a similar LP model and solves the problem (with integer data) in weakly-polynomial time using a binary search algorithm. Levner and Kats [13] and Chen et al. [1] use graph constructions for a representation of the no-wait production process with arbitrary time windows, and modify the Bellman–Ford dynamic programming method. The complexity of the algorithm in [1] is $O(m^4p^2)$ where m is the number of nodes (workstations) and p is the number of arcs in the underlying graph. The algorithm in [13], though exploiting a more laborious node labelling, has a better worst-case complexity, namely $O(m^2p)$. Some authors have investigated a version of the latter problem in which the time intervals are not bounded from above (see [3,5,18,21]). These studies are based on various graph-theoretic representations of the production process; the complexity of the suggested algorithms is $O(m^3)$, m being the number of workstations. Ng and Leung [16] derived mathematical programming models for a version of a robotic problem in which durations of technological operations, as well as durations of loaded robot moves, are considered as decision variables lying in prescribed intervals. Such model was also considered in [15]. This condition makes the problem more flexible though more complicated. They constructed a modification of the binary search algorithm presented in [11], and solved the problem in weakly-polynomial time in the case of integer input data.

In the present paper, the latter problem with flexible, interval-valued operation durations is solved for real input data in polynomial time, and the algorithm is extended to the case of multiple robots. We also use critical path problem ideas to solve these problems (see, by example, [2] and [20] for classical books on PERT/CPM theory; a brief outline of basic definitions is given in Appendix). We further develop the mathematical model and techniques suggested in [13]. Main advantages of our model in comparison with the aforementioned models are the following: (i) we consider a more general case of *multiple* robots in production cells; (ii) all processing, setup and transportation operations are assumed to be flexible and lying in prescribed time windows and (iii) we show that the generalized multi-robot scheduling problem remains polynomial solvable. Notice that if we try to further generalize the problem—for instance, require robots' routes be variable rather than known in advance—then the arising problem becomes NP-hard (see