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Discrete Optimization

The Tree of Hubs Location Problem

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ABSTRACT

This paper presents the Tree of Hubs Location Problem. It is a network hub location problem with single assignment where a fixed number of hubs have to be located, with the particularity that it is required that the hubs are connected by means of a tree. The problem combines several aspects of location, network design and routing problems. Potential applications appear in telecommunications and transportation systems, when set-up costs for links between hubs are so high that full interconnection between hub nodes is prohibitive. We propose an integer programming formulation for the problem. Furthermore, we present some families of valid inequalities that reinforce the formulation and we give an exact separation procedure for them. Finally, we present computational results using the well-known AP and CAB data sets.

1. Introduction

Hub networks have their origin in transportation and telecommunication systems where several origin/destination points send and receive some product. The key feature of these networks is to route products via a specific subset of links, rather than routing each product with a direct link from its origin to its destination point. In particular, hub networks use a set of hub nodes to consolidate and reroute the flows, and a reduced number of links, where economies of scale are applied, to connect the (usually large) set of origins/destination points. Broadly speaking, Hub Location Problems (HLP) consider the location of a set of hub nodes and the design of the hub network.

Two basic HLP models appeared in the literature more than a decade ago: the multiple allocation and the single allocation uncapacitated hub location problems. The first formulation for the multiple allocation case was given by Campbell (1994). Later, Klincewicz (1996), Skorin-Kapov et al. (1996), Ernst and Krishnamoorthy (1998a,b) and Mayer and Wagner (2002) studied several improvements, while Hamacher et al. (2004) and Cánovas et al. (2006) began the polyhedral study of the problem. Recent advances have been presented in Marín (2005b) and Cánovas et al. (2007). The single allocation case has been studied by O'Kelly (1987), Klincewicz (1991), Skorin-Kapov et al. (1996), Aykin (1995) and Ernst and Krishnamoorthy (1998a). The capacitated multiple allocation case was studied by Aykin (1994), Ebery et al. (2000), Campbell (1994), Boland et al. (2004), and by Marín (2005a). The capacitated single allocation problem has also been studied by Ernst and Krishnamoorthy (1999), Labbé et al. (2005), Contreras et al. (2008, 2009), and by Contreras (2009). The interested reader is referred to the comprehensive surveys on the matter Campbell et al. (2002) and Alumur and Kara (2008).

Traditionally, most hub location models assume that hubs are fully interconnected, that is to say, that there exists a link connecting any pair of hubs. However, there exist many applications in which the backbone network (i.e. the network connecting the facilities) is not fully interconnected (see, for instance O'Kelly and Miller, 1994; Klincewicz, 1998). The so-called hub-arc location models Campbell et al. (2005a,b) impose no condition on the structure of the arcs that connect hubs, and they do not even require these arcs to define one single connected component. Moreover, some hub location models have been proposed recently where two hubs are not necessarily connected by some link (see, for instance Nickel et al., 2001; Gelareh, 2008).

In this paper we propose the Tree of Hubs Location Problem (THLP). The THLP is a single allocation hub location problem where p hubs have to be located on a network and connected by means of a (non-directed) tree. Then each non-hub node must be connected (allocated) to a hub and all the flow between nodes must use these connections to circulate, i.e., excepting the arcs that connect each non-hub node with its allocated hub, the arcs that route the flows must be links connecting hubs. There is a per unit transportation cost associated with each arc. The objective is to minimize the operation costs of the system.

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As already stated in Klincewicz (1998) and Campbell et al. (2002) there are some problems in the literature, under slightly different names, dealing with the location of a tree on a network in the context of hub location. In the case of hierarchical networks, Kim and Tcha (1992) presented a problem with a tree-star topological configuration. In the case of digital data service networks, Lee et al. (1994, 1996) consider the problem of locating a set of hubs and connecting them by means of a tree. All these problems have the fixed installation costs as their dominant cost components, while the THLP has the routing or flow cost as its dominant cost component.

In addition to the above problems and to other hub location problems, our problem is also related to other types of problems studied in the literature where location, design or routing decisions are considered. To mention just a few: (a) the location of tree-shaped facilities (also called extensive facilities) on a network, which has been studied within location theory (Hakimi et al., 1993; Kim et al., 1996; Puerto and Tamir, 2005); (b) network design problems that study the arcs to be used in the optimal routing of flows within pairs of nodes on a network, like the Optimum Communication Spanning Tree Problem (Hu, 1974) or the Minimum Sum Violation Tree Problem (Chen et al., 2008); (c) rapid transit network design problems that locate lines in rail transit systems (Marín, 2007); and (d) some location-routing problems, like the ring-star problem (Labbé et al., 2004; Laporte and Rodríguez-Martín, 2007).

Potential applications of models where facilities are connected by means of a tree arise when the cost of the links between facilities is very high, and as a consequence full interconnection is prohibitive. For sending all the flows in the network a path must exist between each pair of facilities, i.e., a connected graph must be built. But due to the high cost of the connections, connectivity must be achieved using the minimum number of links. Specific applications of such problems arise mostly in telecommunications (see Hu, 1974; Nguyen and Knippel, 2007) and in transportation (see, for instance, the recent work of Chen et al. (2008) for an excellent description of the practical relevance of tree-backbone problems in small package delivery). One concrete example of an application of the THLP is the design of the high-speed train network in Spain, which is currently under construction and which is planned to be completed by 2020. This train network has been designed with a tree structure and it is intended that, when finished, each city with more than 10,000 inhabitants will be within 50 km of some high-speed train station. Another application is the design of rapid transit systems for urban areas where there are flows of users (citizens) who must travel between origin/destination pairs. Depending on the criterion used for establishing the allocation of users, these applications would fit in the THLP model.

In this paper, we propose a mixed integer programming formulation for the THLP. This formulation uses two sets of two indices binary variables to model the location/allocation and the design decisions, respectively, and one set of three indices continuous variables to model the flows which are routed through the tree of hubs. Moreover, we present several families of valid inequalities to strengthen the formulation of the proposed model (and, thus, its LP bound) together with their exact separation procedures. We have performed a series of computational experiments in order to assess the quality of the formulation and the strength of the proposed valid inequalities. The obtained numerical results confirm the efficiency of the proposed inequalities since, in all tested instances ranging from 10 to 25 nodes, they are able to considerably reduce both the duality gap and the required computational time to optimally solve them.

The paper is organized as follows. First, we formally define the problem. In Section 3, we present a mathematical programming formulation for the THLP. Later, in Section 4, we derive some families of valid inequalities and separation procedures to strengthen the formulation. The computational results obtained are given in Section 5. The paper finishes with a section of conclusions and some possible future lines of research.

2. Description of the problem

Consider a complete digraph without loops $G = (N,A)$ whose set of nodes, $N = \{1,\ldots,n\}$, represents the set of origins and destinations of a certain product that is routed through G via some transshipment nodes that are called *hubs*. For each pair of nodes $i,j \in N$, let W_{ij} denote the demand of product from i to j. When some of the end-nodes of the arc $(r,s) \in A$ is not a hub, each unit of product that traverses (r,s) incurs a cost $c_{rs} \geqslant 0$, whereas when both r and s are hubs a discount factor $0 \leqslant \alpha \leqslant 1$ is applied, and the per unit cost associated with arc (r,s) is αc_{rs}.

Any node of N can be chosen to become a hub, and there is a fixed number p of nodes of N, $3 \leqslant p \leqslant n-1$, that must be chosen to be hubs. For each pair $i,j \in N$, if neither i nor j are hubs the flow W_{ij} must go from i to j through one or more intermediate hubs. When i or j are hubs W_{ij} can go directly from i to j, although it is also possible that W_{ij} uses one or more intermediate hubs. Non-hub nodes cannot be used to transship the product. We also require *single allocation* and, thus, every non-hub node i must be assigned (allocated) to one single hub, so that all the flow from/to i to/from any other node must pass through this hub.

We require the hubs to be connected by means of a (non-directed) tree. Observe that since the links that connect hubs are required to define a tree (that we denote *small tree*), the structure that results after adding to the small tree the links connecting non-hub nodes to their allocated hub nodes defines a tree in G (that we denote *large tree*). Thus, in the large tree the path between each pair i and j will be unique. In this path all the transshipment nodes will be hubs so all the arcs, excepting (possibly) the first and last ones, link hubs.

The cost per unit of flow of the path between a pair i and j is the sum of the costs of the arcs in the path, where the discount factor is applied to all the inter-hubs links. Therefore, if i and j are both hubs, W_{ij} could be sent from i to j (directly, if there is such an arc in the small tree, or through additional intermediate hubs, otherwise) and the discount factor will be applied to all the arcs in the route. If either i or j are hubs, the product could be sent directly (if such an arc is part of the large tree) without discount, or through some other hubs, otherwise; if i (resp. j) was the hub, the discount will be applied to all the arcs except the last (resp. first) one. If neither i nor j are hubs, W_{ij} will again be sent from i to j through at least one intermediate hub. Now, W_{ij} must go from i to some hub, then possibly to some other hub(s), and finally to j, and the discount factor will be applied to all the intermediate arcs, but will not be applied neither to the first nor to the last arcs. Note that there can be a flow of product W_{ii} with origin and destination at the same node i. We assume free self service, i.e., $c_{ii} = 0$ $\forall i \in N$. Thus, if i is a hub, W_{ii} can stay in i at null cost, whereas if i is not a hub, W_{ii} must go to some hub before returning to i. Note also that triangle inequality of costs c_{ij} is not assumed. Some formulations in the literature (see Cánovas et al., 2006) are only valid if this property holds, but our formulation is also valid without this assumption. We do not assume symmetry either, i.e., costs c_{ij} and c_{ji} can be different.

We define the *graph of flows*, G_F, as the undirected graph with vertex set N and an edge associated with each pair $(i,j) \in N \times N$ such that $W_{ij} + W_{ji} > 0$. We assume that G_F has one single connected component since, otherwise, it would be more natural to define as many hub