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Discrete Optimization

A branch and bound algorithm for scheduling trains in a railway network

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Abstract

The paper studies a train scheduling problem faced by railway infrastructure managers during real-time traffic control. When train operations are perturbed, a new conflict-free timetable of feasible arrival and departure times needs to be re-computed, such that the deviation from the original one is minimized. The problem can be viewed as a huge job shop scheduling problem with no-store constraints. We make use of a careful estimation of time separation among trains, and model the scheduling problem with an alternative graph formulation. We develop a branch and bound algorithm which includes implication rules enabling to speed up the computation. An experimental study, based on a bottleneck area of the Dutch rail network, shows that a truncated version of the algorithm provides proven optimal or near optimal solutions within short time limits.

Keywords: Train scheduling; Real-time conflict resolution; Alternative graph; Branch and bound algorithm

1. Introduction

Railway operators plan train services in detail, defining several months in advance the train order and timing at crossings, junctions and platform tracks. A robust timetable is able to deal with minor delays occurring in real-time. However, technical failures and disturbances may influence the running times, dwelling and departing events, thus causing primary delays. Due to the interaction between trains, these delays may be propagated as secondary delays to other trains in the network. A partial modification of the timetable may then be required during operations. Hence, managing railway traffic in real-time requires re-scheduling train movements through the network, minimizing secondary delays and ensuring the feasibility of the resulting plan of operations. This process requires effective solutions within minutes and is called train dispatching, or *conflict resolution*.

To a large extent, conflict resolution issues are solved by human operators, called dispatchers, each controlling a single dispatching area. Elementary

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Decision Support Systems have been developed to help them to quickly and effectively re-schedule train movements, but the usual policy still consists of scheduling trains following the order in the timetable or according to commonly adopted dispatching rules. For example, practical applications of local simple measures for the Dutch railway network, based on train re-ordering at crossing points on a first come first served basis, are described in [11,24]. However, we believe there is a need for more effective conflict resolution systems which are able to exploit global information about the status of the network. In this paper, we therefore address the real-time conflict resolution problem in the development of a detailed optimization model and a branch and bound procedure. The model and algorithms discussed here may also have other applications, e.g., as planning tools to validate the impact of possible timetable perturbations.

There has been increasing interest on train scheduling and timetabling problems since the pioneering paper of Szpigel [25], in which the problem of scheduling trains on a single track line was modeled as a job shop scheduling problem and solved with a branch and bound algorithm. Cordeau et al. [7] present a comprehensive survey on train scheduling and routing problems. In view of the extensiveness of their review, we provide only a brief summary here.

At least two main research lines can be identified in this context, the timetable design problem and the real-time conflict resolution problem. The former case includes a number of off-line problems at various levels of detail, and typically focuses on large rail networks and simplified models. For example, the track line between two stations is typically represented as a single resource, and it is possible to cancel potential trains from the schedule when the track capacity is insufficient. Most authors address this problem on a line with single and/or multiple track segments [25,17,15,2,20,3].

The research on real-time conflict resolution systems aims to support the dispatchers in restoring the schedules' feasibility, given the real-time positions of the trains. In order to provide physically feasible solutions with short computation times, of particular interest is a limited time horizon and a detailed representation of the area managed by a single dispatcher. Hence, the basic resource is the track line between two signals, and it is possible to cancel some train journeys by solely assigning an alternative trip to the vehicle. In the few recent papers addressing the conflict resolution problem, Higgins et al. [12] develop a branch and bound method, while Şahin [23] and Higgins et al. [13] develop heuristic algorithms for re-scheduling trains on a railway line by modifying crossing and overtaking in case of conflicts. Adenso Diaz et al. [1] described a real-time dispatching system on a regional network composed of a single corridor with three major lines. Fay [10] suggested an expert system using Petri Nets and a fuzzy rule-base for train traffic control during disturbances and performed experiments on small fictional instances. Ping et al. [22] proposed a method based on genetic algorithms for adjusting the train orders and times on a double-track line. Dorfman and Medanic [9] propose a discrete-event model for scheduling trains on a single line and a greedy strategy to obtain sub-optimal schedules. Dessouky et al. [8] develop a detailed MILP formulation and a branch and bound procedure for determining the optimal dispatching times for the trains traveling in a complex rail network.

Our work is concerned with the real time conflict resolution problem on a regional rail network. Our optimization model incorporates a detailed description of the network topology, including railway signal aspects and safety rules, on the basis of each track segment between two block signals being able to host at most one train at a time. We specifically model train trips as jobs to be scheduled on track segments, which are viewed as machines, as in Szpigel [25], Şahin [23], and Oliveira and Smith [20]. However, differing from other authors, we explicitly include no-store constraints in the optimization model, which requires that a train, having reached the end of a track segment, cannot enter the subsequent segment if the latter is occupied by another train, thus preventing other trains from entering the former segment. To achieve this, we use the alternative graph formulation of Mascis and Pacciarelli [18], which allows modeling job shop scheduling problems with no-wait and no-store constraints. We also show that the alternative graph model can easily incorporate several other railway relevant traffic regulation rules and constraints, which are rarely taken into account in the literature, as observed by Oliveira and Smith [20]. Examples of such aspects include speed restrictions, precedence and meeting constraints.

We describe a specific branch and bound procedure to solve the conflict resolution problem. It is worthwhile noting that the alternative graph formulation of a practical size instance may include hun-